CHAPTER 8
VARIABLES CONTROL CHARTS

Sections

Chapter Objectives

- To distinguish between attribute data and variables data
- To discuss variables charts and the PDSA cycle
- To discuss the determination of subgroup size and the frequency of subgroup selection
- To discuss and illustrate the construction and interpretation of $\bar{x}$ and R charts
- To discuss and illustrate the construction and interpretation of $\bar{x}$ and s charts
- To discuss and illustrate the construction and interpretation of individuals and moving range charts
- To discuss the conditions for revision of control limits for variables charts
- To discuss rational subgrouping of data

8.1 Introduction

Variables (measurement) data consist of numerical measurements such as cycle time, waiting time, revenue, cost, weight, length, temperature, and electrical resistance. Variables data contains more information than attribute data because it measures a characteristic over a continuous scale (as opposed to discrete measurements, such as conforming or non-conforming), or it measures the difference between a numerical characteristic and its desired (nominal) value, even within specification limits. Measurement data can be used to reduce variation in a process, even if all units of output are within specification limits. This is the Taguchi Loss Function view of quality, as shown in Figure 1.15 in Chapter 1; it results in the never-ending reduction of variation in a process. Measurement data use all the information contained in the data; this alone makes variables charts preferable to attribute charts when a choice is possible.

There are three principal types of variables control charts: the $\bar{x}$ and R chart, the $\bar{x}$ and s chart, and the individuals and moving range chart. All are used in the never-ending spiral of process improvement.

As the proportion of defective output decreases, or the number of defects per unit decreases, attribute control charts require very large subgroup sizes to detect the rare defectives or defects. The only way to overcome the need for larger and larger subgroup sizes is to continue upward on the spiral of quality consciousness through the use of variables control charts.

8.2 Variables Charts and the PDSA Cycle

The **PDSA cycle** provides guidelines for proceeding with variables control charts to continually reduce unit-to-unit variation, as demonstrated by the Taguchi Loss Function.

8.2.1 Plan

The purpose of the control chart must be carefully delineated to effectively use it as a vehicle to reduce the difference between customer needs and process performance. A plan must be established that clearly shows what will be control charted, why it will be control charted, where it will be control charted, when it will be control charted, who will do the control charting, and how it will be control charted. Consequently, organization personnel must decide which metrics to measure. These decisions require the cooperation and input of all those directly or indirectly involved with the process, including operators, foremen, supervisors, and engineers. Some of the techniques discussed in Chapter 10, such as brainstorming, Cause and Effect diagrams, check sheets, and Pareto diagrams, may be useful in determining the process metrics that will decrease the difference between customer needs and process performance. Often some feature of the process which has been a source of trouble (resulting in extra cost in scrap or rework) and has failed to yield to corrective efforts is a good place to start. Starting far upstream, or near the beginning of a process, will often produce the most dramatic results and may offer the greatest opportunity to alter the factors at the root of downstream special causes of variation.

Operational definitions, as discussed in Chapter 4, must be constructed and communicated to people involved with the data collection. At this point any forms to be used for data collection should be selected or designed and the responsibility for construction of the charts assigned.

8.2.2 Do

Data collection and the calculation of control chart statistics constitute the Do stage for constructing variables control charts. It is usually best to collect at least 20 subgroups before beginning to construct a control chart. On rare occasions, fewer than 20 subgroups may be used, but a control chart should almost never be attempted with fewer than 10 subgroups.

Variables charts consist of are made up of two parts: one charts the **process variability**, and one charts the **process location**. For instance, in the $\bar{x}$ and R chart, the R values, or subgroup ranges, are used to track the variability of a process. The $\bar{x}$ values, or subgroup averages, are used to track the location of a process. As the control limits of the portion of the control chart measuring location are based on the average variability ($\bar{R}$ in the $\bar{x}$ and R chart), the variability-measuring portion of the control chart must be constructed and evaluated first. Only if there is stability in the variability portion of the control chart can the location portion of the control chart be constructed. For each of the variables control charts, the estimate of the process standard deviation is based on the average value of the measure of variability used for the subgroups. If the process is not stable, its variability will not be predictable, leading to unreliable estimates of the process standard deviation. If these estimates are used to construct control limits, those limits will also be unreliable and may not reveal special sources of variation when they exist.

After the initial data set has been collected, the centerline, control limits, and zone boundaries (if applicable) should be computed for both portions of the control chart. These should be entered onto the control charts along with the collected set of data points. First, the variability portion of the control chart should be examined for indications of special variation. Any special causes of variation must be studied and the process must be stabilized before the location portion of the control chart is analyzed. This means skipping ahead to the Study stage of the PDSA cycle before completing the Do stage.

If there are no indications of a lack of control in the control chart's variability portion, the location portion of the chart can be analyzed in the Study stage of the PDSA cycle. This completes the Do stage.

8.2.3 Study

Indications of a lack of control, such as patterns of the type introduced in Chapter 6, are identified in the Study stage of the PDSA cycle. These indications of special sources of variation may be found in the chart dealing with variation, in the chart dealing with location, or in both. Whether the variation is common process variation or special process variation, once we have found and identified the sources, we proceed to the Act stage to set policy to formalize process improvements resulting from analysis of the control chart.

We must periodically review all aspects of the control chart and make changes where appropriate. If the process itself has been changed in some way, the control limits should certainly be recomputed and the analysis of the process begun anew.

8.2.4 Act

If the variation found in the Study stage results only from common causes, then efforts to reduce that variation must focus on changes to the process itself. When indications of special causes of variation are present, the cause or causes of that special variation should be removed if the variation is detrimental or incorporated into the process if the variation is beneficial.

The focus of the Act stage is on formalizing policy that results directly from the prior study of the causes of process variation. This will lead to a reduction in the difference between customer needs and process performance.

Last, the purpose of the control chart must be reconsidered by returning to the Plan stage. This will help to maintain a focus on improvement and on those areas that can be most beneficial in reducing the difference between customer needs and process performance.

8.3 Subgroup Size and Frequency

The construction of an appropriate control chart depends, in part, on the **subgroup size**. Many factors affect the ideal subgroup size. Subgroups should be large enough to detect data points or data patterns indicating a lack of control, when a lack of control exists. Both statistical expertise and process expertise are required to determine the proper subgroup size for a control chart.

Individuals and moving range charts are used when only one variables measurement is available or appropriate as a subgroup; for example, monthly labor expense or hours per week devoted to repair calls. $\bar{x}$ and Range charts (**$\bar{x}$ and R charts**) are used with a subgroup size of two through nine. $\bar{x}$ and Standard Deviation charts (**$\bar{x}$ and s charts**) are generally used with subgroup sizes of ten or more.

Subgroup frequency, or how often subgroups are selected, depends on the particular application. If a process can change very quickly, then subgroups should be drawn more frequently than the process is expected to change, based upon the opinion of a process expert. This helps prevent missing changes to the process.

8.4 $\bar{x}$ and R Charts

As the name implies, the $\bar{x}$ and R chart uses the subgroup range, R, to chart the process variability (frequently over time), and the subgroup average, $\bar{x}$, to chart the process location (frequently over time). Stable processes yield subgroups that will behave predictably, enabling us to construct an $\bar{x}$ and R chart. The two characteristics, $\bar{x}$ and R, can be estimated by relatively simple procedures. Estimates of the standard errors of both $\bar{x}$ and R are based on the average subgroup range, $\bar{R}$.

8.4.1 An Example

A large pharmaceutical firm provides vials filled to a nominal value of 52.0 grams. The firm's management has embarked on a program of statistical process control and has decided to use variables control charts to detect special causes of variation in this filling process. Samples of six vials are selected every five minutes during a 105-minute period. The subgroup size and subgroup frequency were jointly determined by a process expert and a statistician according to the guidelines discussed above. Each set of six measurements makes up a subgroup.

The appropriate control chart in this instance is an $\bar{x}$ and R chart since there are six variables measurements per subgroup. The purpose of this chart is to see whether the process output is stable with regard to its variability and its average value. The output of each subgroup is summarized by its sample average and range. $\bar{x}$ is the average for each of the subgroups, as given by Equation (5.3), while R is the range, as calculated using Equation (5.8).

The Range Portion.

For our example, the subgroup ranges are shown in Table 8.1. They begin with a range at 9:30 of

$$R = 53.10 - 52.22 = 0.88$$

and continue for all 22 subgroups to the last range at 11:15 of

$$R = 52.16 - 51.67 = 0.49$$

Table 8.1

Vial Weights

Obs. No.	Time	Measurement (grams) 1	2	3	4	5	6	R	
1	9:30	52.22	52.85	52.41	52.55	53.10	52.47	0.88	52.60
2	9:35	52.25	52.14	51.79	52.18	52.26	51.94	0.47	52.09
3	9:40	52.37	52.69	52.26	52.53	52.34	52.81	0.55	52.50
4	9:45	52.46	52.32	52.34	52.08	52.07	52.07	0.39	52.22
5	9:50	52.06	52.35	51.85	52.02	52.30	52.20	0.50	52.13

6	9:55	52.59	51.79	52.20	51.90	51.88	52.83	1.04	52.20
7	10:00	51.82	52.12	52.47	51.82	52.49	52.60	0.78	52.22
8	10:05	52.51	52.80	52.00	52.47	51.91	51.74	1.06	52.24
9	10:10	52.13	52.26	52.00	51.89	52.11	52.27	0.38	52.11
10	10:15	51.18	52.31	51.24	51.59	51.46	51.47	1.13	51.54
11	10:20	51.74	52.23	52.23	51.70	52.12	52.12	0.53	52.02
12	10:25	52.38	52.20	52.06	52.08	52.10	52.01	0.37	52.14
13	10:30	51.68	52.06	51.90	51.78	51.85	51.40	0.66	51.78
14	10:35	51.84	52.15	52.18	52.07	52.22	51.78	0.44	52.04
15	10:40	51.98	52.31	51.71	51.97	52.11	52.10	0.60	52.03
16	10:45	52.32	52.43	53.00	52.26	52.15	52.36	0.85	52.42
17	10:50	51.92	52.67	52.80	52.89	52.56	52.23	0.97	52.51
18	10:55	51.94	51.96	52.73	52.72	51.94	52.99	1.05	52.38
19	11:00	51.39	51.59	52.44	51.94	51.39	51.67	1.05	51.74
20	11:05	51.55	51.77	52.41	52.32	51.22	52.04	1.19	51.89
21	11:10	51.97	51.52	51.48	52.35	51.45	52.19	0.90	51.83
22	11:15	52.15	51.67	51.67	52.16	52.07	51.81	0.49	51.92

The average of the R values is called $\overline{R}$, and it is computed by taking the simple arithmetic average of the R values:

$$\overline{R} = \frac{\sum R}{k} \quad (8.1)$$

where k is the number of subgroups.

Thus, the centerline can be computed as

$$\overline{R} = \frac{16.28}{22} = 0.74$$

Using a multiple of three standard errors to construct the upper control limit and the lower control limit, we have

$$\begin{aligned} UCL(R) &= \overline{R} + 3\sigma_R \\ LCL(R) &= \overline{R} - 3\sigma_R \quad (8.2) \end{aligned}$$

Assuming that the distribution of process output was, and will be, stable and approximately normally distributed, we can derive control limits. (Note that from the **Empirical Rule** discussed in Chapter 5, the assumption of normality is not necessary to interpret the control limits.)

When the characteristic of interest is stable and approximately normally distributed, and the subgroup size is small ($2 \le n \le 9$), then the relationship between the range and the process standard deviation is:

$$\overline{R} = d_2\sigma \quad (8.3)$$

and

$$\sigma_R = d_3\sigma = d_3[\overline{R}/d_2] \quad (8.4)$$

where σ is the process standard deviation and the values of d_2 and d_3 are tabulated as a function of subgroup size in Table B.1 in Appendix B.

We can thus replace $\overline{R}$ and σ_R with these functions of the process standard deviation, σ:

$$UCL(R) = d_2\sigma + 3d_3\sigma = d_2\sigma\left(1 + \frac{3d_3}{d_2}\right)$$

$$= \overline{R}\left(1 + \frac{3d_3}{d_2}\right) \quad (8.5)$$

Similarly,

$$LCL(R) = d_2\sigma - 3d_3\sigma = d_2\sigma\left(1 - \frac{3d_3}{d_2}\right)$$

$$= \overline{R}\left(1 - \frac{3d_3}{d_2}\right) \quad (8.6)$$

A more convenient way to represent these control limits is by defining two new constants, D_3 and D_4:

$$D_3 = 1 - \frac{3d_3}{d_2} \quad (8.7)$$

$$D_4 = 1 + \frac{3d_3}{d_2} \quad (8.8)$$

So that

$$UCL(R) = D_4\overline{R} \quad (8.9)$$

and

$$LCL(R) = D_3\overline{R} \quad (8.10)$$

where D_3 and D_4 are tabulated as a function of subgroup size in Table B.1 in Appendix B. For our example

$$UCL(R) = 2.004(0.740) = 1.48$$

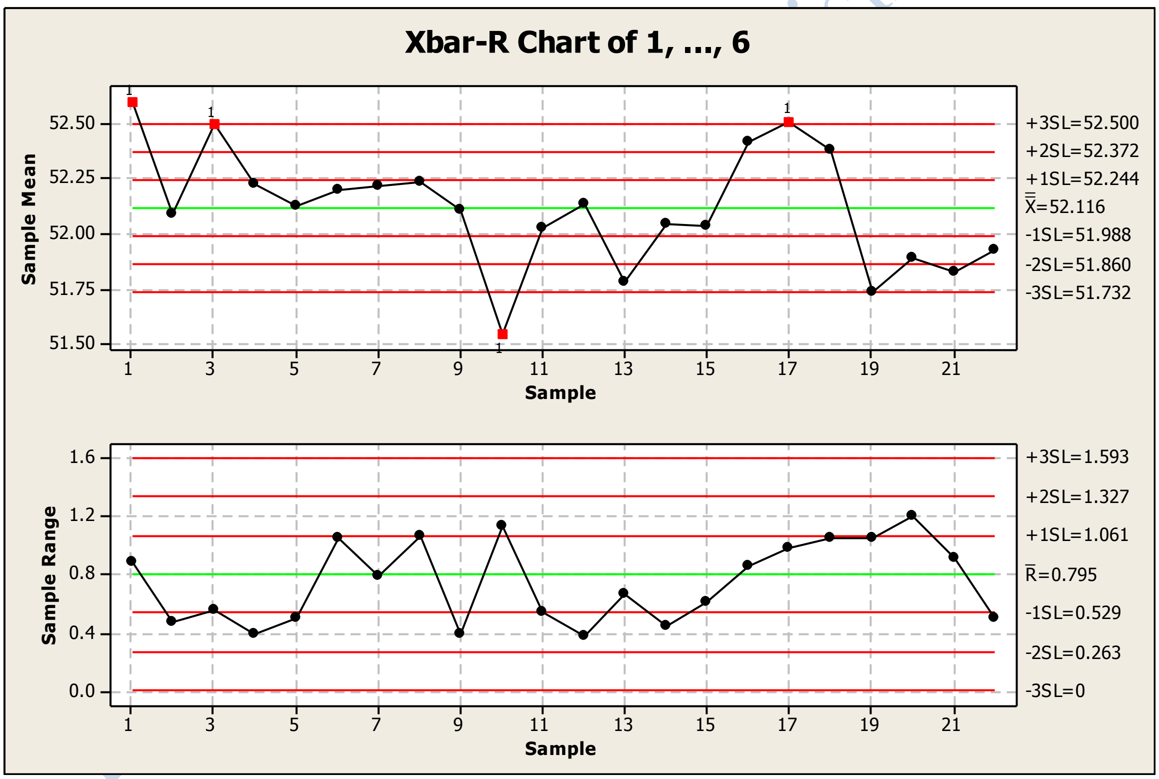
and

$$LCL(R) = 0(0.740) = 0$$

The bottom portion of Figure 8.1 illustrates the resulting average range and upper and lower control limits for the subgroup ranges. The R chart is then examined for signs of special variation. None of the points on the R chart is outside of the control limits, and there are no other signals indicating a lack of control. Thus, there are no indications of special sources of variation on the R chart. More will be said later in this chapter and in Chapter 9 about indications of special causes of variation.

Figure 8.1

$\bar{x}$ and R Charts for Filling Operations



In the determination of the upper and lower control limits, three times the estimated standard error of the subgroup ranges, σ_R, was added to and subtracted from the average range value, $\bar{R}$, yielding the factors D_4 and D_3. Recall from Chapter 6 that the **zone boundaries** for zones A, B, and C are positioned at one and two standard errors on either side of the control chart centerline. Using the estimated standard error of $\bar{R}$ from Equation (8.4), the zone boundaries are given by

$$\text{Boundary between lower zones A and B} = \bar{R} - 2d_3(\bar{R}/d_2) = \bar{R}(1 - 2d_3/d_2) \qquad (8.11)$$

Boundary between lower zones B and C = $\bar{R} - 1d_3(\bar{R}/d_2) = \bar{R}(1 - d_3/d_2)$ (8.12)

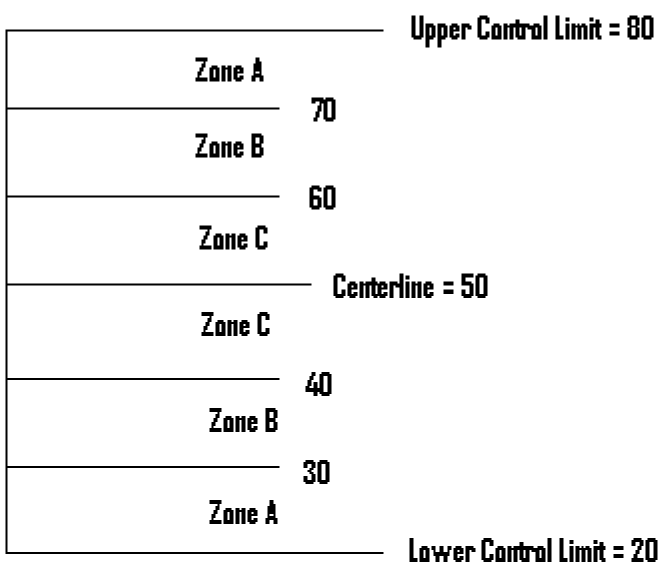
When the result of Equations (8.11) or (8.12) is a negative number, 0.00 is used instead, as negative ranges are meaningless. Also,

Boundary between upper zones B and C = $\bar{R} + d_3(\bar{R}/d2) = \bar{R}(1 + d_3/d_2)$ (8.13)

Boundary between upper zones A and B = $\bar{R} + 2d_3(\bar{R}/d_2) = \bar{R}(1 + 2d_3/d_2)$ (8.14)

An easy way to position the zone boundaries is to divide the distance between the upper control limit and the centerline by 3. The resulting quantity is then added to and subtracted from the centerline to form the boundaries between the C and B zones. The boundaries between the B and A zones are formed by adding and subtracting twice the result of dividing the distance between the upper control limit and the centerline by 3. For example, if the average range were 50 and the control limits were 20 and 80, the distance between the upper control limit and the centerline would be 30. The resulting zone boundaries are shown in Figure 8.2. Assuming that the range portion of the control chart is stable, the $\bar{x}$ portion may now be developed.

Figure 8.2
A, B and C Zones for a Control Chart



The $\bar{x}$ Portion.

After analyzing the R chart, we may analyze the $\bar{x}$ chart. This control chart depicts variations in the averages of the subgroups. To find the average for each subgroup, we add the data points for each subgroup and divide by the number of entries in the subgroup, as given by Equation (5.3). For the pharmaceutical company, the average of the 9:30 subgroup is

$$\frac{52.22+52.85+52.41+52.55+53.10+52.47}{6}=52.60$$

This calculation is repeated for each of the subgroups. The $\bar{x}$ results for this example can be found in the last column of Table 8.1.

The centerline of an $\bar{x}$ control chart is found by taking the average of the subgroup averages, $\bar{\bar{x}}$, calculated from Equation (5.4). In our example, the average of the 22 subgroup averages is

$$\bar{\bar{x}}=\frac{1146.55}{22}=52.12$$

Using a multiple of three standard errors to construct the control limits, we have

$$UCL(\bar{x})=\bar{\bar{x}}+3\sigma_{\bar{x}}$$

$$LCL(\bar{x})=\bar{\bar{x}}-3\sigma_{\bar{x}} \qquad (8.15)$$

Assuming that the distribution of process output was, and will be, stable and approximately normally distributed, we can derive the control limits. (Note that due to the Empirical Rule discussed in Chapter 5, the assumption of normality is not necessary to interpret the control limits.)

Consequently, when the characteristic of interest is stable and approximately normally distributed,

$$\bar{\bar{x}}=\frac{\sum\bar{x}}{k} \qquad (8.16)$$

where k is the number of subgoups.

$$\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}=\frac{\bar{R}/d_2}{\sqrt{n}} \qquad (8.17)$$

that is, since $\overline{R} = d_2\sigma$, $\sigma = \dfrac{\overline{R}}{d_2}$; the values of d_2 are tabulated as a function of subgroup size in Table B.1 in Appendix B.

We can thus rewrite the control limits as

$$UCL(\overline{x}) = \overline{\overline{x}} + 3\left[\frac{\overline{R}/d_2}{\sqrt{n}}\right] \qquad (8.18)$$

or,

$$UCL(\overline{x}) = \overline{\overline{x}} + A_2\overline{R} \qquad (8.19)$$

and,

$$\begin{aligned} LCL(\overline{x}) &= \overline{\overline{x}} - 3\sigma_{\overline{x}} \\ &= \overline{\overline{x}} - 3\frac{\sigma}{\sqrt{n}} \qquad (8.20) \\ &= x - 3\left[\frac{\overline{R}/d_2}{\sqrt{n}}\right] \end{aligned}$$

or

$$LCL(\overline{x}) = \overline{\overline{x}} - A_2\overline{R} \qquad (8.21)$$

where, $A_2 = 3/\left(d_2\sqrt{n}\right)$ and is tabulated as a function of subgroup size in Table B.1 in Appendix B.

For the pharmaceutical company, the upper and lower control limits can now be computed as:

$UCL(\overline{x}) = 52.12 + (0.483)(0.740) = 52.47$

and

$LCL(\overline{x}) = 52.12 - (0.483)(0.740) = 51.76$

The upper portion of Figure 8.1 illustrates the $\overline{x}$ chart. Notice that a total of five points on the $\overline{x}$ chart are outside of the control limits and therefore indicate a lack of control. Further investigation is warranted to determine the source(s) of these special variations.

It is important to recognize that an $\bar{x}$ chart cannot be meaningfully analyzed if its corresponding R chart is not in statistical control. This is because $\bar{x}$ chart control limits are calculated from $\bar{R}$ (i.e., $\bar{\bar{x}} \pm A_2\bar{R}$), and if the range is not stable, no calculations based on it will be accurate.

The product $A_2\bar{R}$ represents three times the standard error of the subgroup means. This is useful in forming the A, B, and C zones used in this chart, as well as in helping to detect patterns indicating a lack of statistical control. Zone boundaries are placed on both sides of the centerline at a distance of one and two times the standard error, respectively. Equivalently, a simple way to position the zone boundaries is to divide the distance between the upper control limit and the centerline by 3. Just as with the range portion, the resulting quantity is then added to and subtracted from the centerline to form the boundaries between the C and B zones. As before, the boundaries between the B and A zones are formed by adding (or subtracting) twice the result of dividing the distance between the upper control limit and the centerline by 3.

$$\text{Boundary between lower zones A and B} = \bar{\bar{x}} - \left(\tfrac{2}{3}\right)A_2\bar{R} \qquad (8.22)$$

$$\text{Boundary between lower zones B and C} = \bar{\bar{x}} - \left(\tfrac{1}{3}\right)A_2\bar{R} \qquad (8.23)$$

$$\text{Boundary between upper zones B and C} = \bar{\bar{x}} + \left(\tfrac{1}{3}\right)A_2\bar{R} \qquad (8.24)$$

$$\text{Boundary between upper zones A and B} = \bar{\bar{x}} + \left(\tfrac{2}{3}\right)A_2\bar{R} \qquad (8.25)$$

8.4.2 Another Example

Consider the case of a manufacturer of circuit boards for personal computers. Various components are to be mounted on each board and the boards eventually slipped into slots in a chassis. The boards' overall length is crucial to assure a proper fit, and this dimension has been targeted as an important item to be stabilized. (Note: the width, thickness, hardness, or any other characteristic may also be targeted either simultaneously or at another point in time.) Boards are cut from large sheets of material by a single rotary cutter continuously fed from a hopper. At a customer's request, it was decided to create a control chart for the length of circuit boards produced by the process.

After input from many individuals involved with the process, as well as a statistician, it is decided to select the first five units every hour from the production output. Each group of five items represents a subgroup. This manner of subgroup selection is most likely to isolate the variation over time between the subgroups and, therefore, capture only common process variation within the subgroups.

Boards are measured using an operationally defined method (BOARD**)**; Table 8.2 lists the resulting lengths. The average and the range for each subgroup of five elements in Table 8.2 have been computed and are shown in the last two columns on the right. This arrangement of the data will be used to determine whether special sources of variation between subgroups are evident in measurement-to-measurement changes over time.

Table 8.2

Cut Circuit Board Lengths

Time	Sample Number	1	2	3	4	5	Average $\bar{x}$	Range R
9:00 AM	1	5.030	5.002	5.019	4.992	5.008	5.010	0.038
10:00 AM	2	4.995	4.992	5.001	5.011	5.004	5.001	0.019
11:00 AM	3	4.988	5.024	5.021	5.005	5.002	5.008	0.036
12:00 PM	4	5.002	4.996	4.993	5.015	5.009	5.003	0.022
1:00 PM	5	4.992	5.007	5.015	4.989	5.014	5.003	0.026
2:00 PM	6	5.009	4.994	4.997	4.985	4.993	4.996	0.024
3:00 PM	7	4.995	5.006	4.994	5.000	5.005	5.000	0.012
4:00 PM	8	4.985	5.003	4.993	5.015	4.988	4.997	0.030
5:00 PM	9	5.008	4.995	5.009	5.009	5.005	5.005	0.014
6:00 PM	10	4.998	5.000	4.990	5.007	4.995	4.998	0.017
7:00 PM	11	4.994	4.998	4.994	4.995	4.990	4.994	0.008
8:00 PM	12	5.004	5.000	5.007	5.000	4.996	5.001	0.011
9:00 PM	13	4.983	5.002	4.998	4.997	5.012	4.998	0.029
10:00 PM	14	5.006	4.967	4.994	5.000	4.984	4.990	0.039
11:00 PM	15	5.012	5.014	4.998	4.999	5.007	5.006	0.016
12:00 AM	16	5.000	4.984	5.005	4.998	4.996	4.997	0.021
1:00 AM	17	4.994	5.012	4.986	5.005	5.007	5.001	0.026
2:00 AM	18	5.006	5.010	5.018	5.003	5.000	5.007	0.018
3:00 AM	19	4.984	5.002	5.003	5.005	4.997	4.998	0.021
4:00 AM	20	5.000	5.010	5.013	5.020	5.003	5.009	0.020
5:00 AM	21	4.988	5.001	5.009	5.005	4.996	5.000	0.021
6:00 AM	22	5.004	4.999	4.990	5.006	5.009	5.002	0.019
7:00 AM	23	5.010	4.989	4.990	5.009	5.014	5.002	0.025
8:00 AM	24	5.015	5.008	4.993	5.000	5.010	5.005	0.022
9:00 AM	25	4.982	4.984	4.995	5.017	5.013	4.998	0.035

Totals 125.030 0.569

Construction of the control chart begins with the range portion. The centerline is found by taking the average of the subgroup ranges using Equation (8.1):

$$\overline{R} = 0.569/25 = 0.023$$

Not only does this value form the centerline for the range portion of the control chart, it also forms the basis for estimating the standard error and hence the control limits and zone boundaries as well. Values for D_3 and D_4, from Table B.1 in Appendix B, are 0.00 and 2.114, respectively. Equations (8.9) and (8.10) yield the control limits:

$$UCL(R) = (2.114)(0.023) = 0.049$$

and

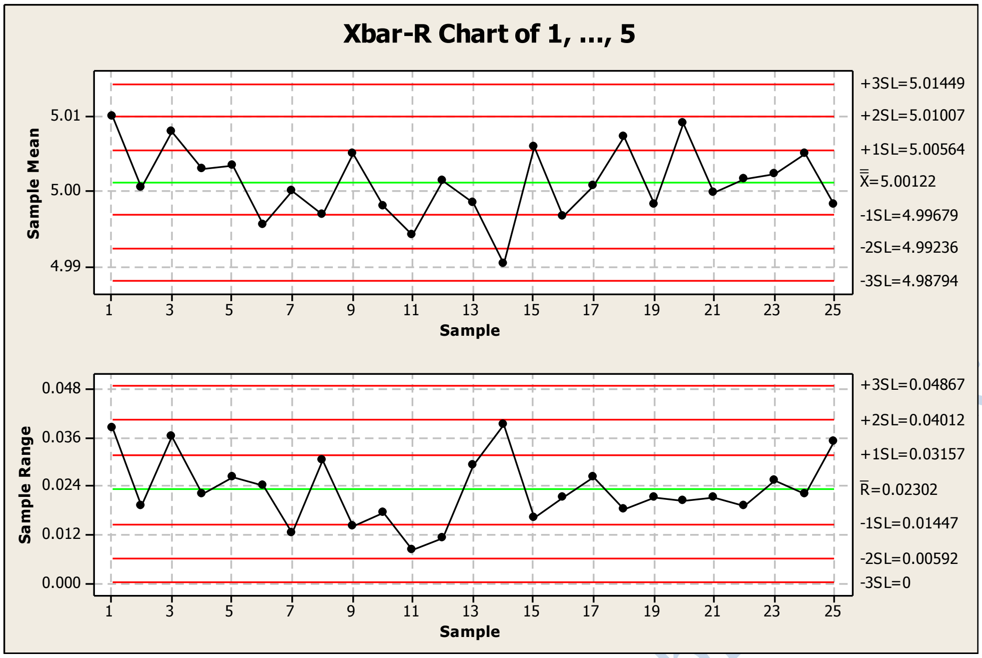
$$LCL(R) = (0)(0.023) = 0.000$$

Equations (8.11), (8.12), (8.13), and (8.14) yield values for the zone boundaries:

Boundary between lower zones A and B = 0.023[1 - 2(0.864)/2.326] = 0.006
Boundary between lower zones B and C = 0.023[1 - 1(0.864)/2.326] = 0.014
Boundary between upper zones B and C =0.023[1 + 1(0.864)/2.326] = 0.032
Boundary between upper zones A and B =0.023[1 + 2(0.864)/2.326] = 0.040

The lower portion of Figure 8.3 displays the control chart. There are no indications of any special sources of variation, and the process appears stable with regard to its range.

Figure 8.3
$\overline{x}$ and R Chart for Circuit Board Length



Once the stability of the range has been established, the $\bar{x}$ portion of the control chart may be constructed. $\bar{R}$ can be used as a basis for the estimate of the standard error for the $\bar{x}$ portion of the chart if the range is stable. Equation (8.16) yields a centerline of

$$\bar{\bar{x}} = 125.029/25 = 5.001$$

The upper and lower control limits can be found using Equations (8.19) and (8.21). The value for A2 comes from Table B.1 in Appendix B for a subgroup of size 5.

$$UCL(\bar{x}) = 5.001 + (0.577)(0.023) = 5.014$$
$$LCL(\bar{x}) = 5.001 - (0.577)(0.023) = 4.988$$

Zone boundaries are found using Equations (8.22 through (8.25):

Boundary between lower zones A and B = 5.001 - (2/3)(0.577)(0.023) = 4.992
Boundary between lower zones B and C = 5.001 - (1/3)(0.577)(0.023) = 4.997
Boundary between upper zones B and C = 5.001 + (1/3)(0.577)(0.023) = 5.005
Boundary between upper zones A and B = 5.001 + (2/3)(0.577)(0.023) = 5.010

These values are shown in the upper portion of Figure 8.3. There are no indications of any special sources of variation present between the subgroups with respect to time in the $\bar{x}$ portion, so we can conclude that this process is stable and its output is predictable in the near future, assuming continued stability.

8.4.3 Another Example

A manufacturer of high-end audio components buys metal tuning knobs to use in assembling its products. Knobs are produced automatically by a subcontractor using a single machine that is designed to produce them with a constant diameter. Because of persistent final assembly problems with the knobs, management has decided to examine this process output by requesting that the subcontractor construct an $\bar{x}$ and R chart for knob diameter. Beginning at 8:30 am on a Tuesday, the first four knobs are selected every half hour. The diameter of each is carefully measured using an operationally-defined technique. The average and range for each subgroup are computed; the data, along with these statistics, are shown in Table 8.3.

Table 8.3 Tuning Knob Diameters

Time	Sample Number	1	2	3	4	Average $\bar{x}$	Range R
8:30 AM	1	836	846	840	839	840.25	10
9:00 AM	2	842	836	839	837	838.50	6
9:30 AM	3	839	841	839	844	840.75	5
10:00 AM	4	840	836	837	839	838.00	4
10:30 AM	5	838	844	838	842	840.50	6
11:00 AM	6	838	842	837	843	840.00	6
11:30 AM	7	842	839	840	842	840.75	3
12:00 PM	8	840	842	844	836	840.50	8
12:30 PM	9	842	841	837	837	839.25	5
1:00 PM	10	846	846	846	845	845.75	1
1:30 PM	11	849	846	848	844	846.75	5
2:00 PM	12	845	844	848	846	845.75	4
2:30 PM	13	847	845	846	846	846.00	2
3:00 PM	14	839	840	841	838	839.50	3
3:30 PM	15	840	839	839	840	839.50	1
4:00 PM	16	842	839	841	837	839.75	5
4:30 PM	17	841	845	839	839	841.00	6
5:00 PM	18	841	841	836	843	840.25	7
5:30 PM	19	845	842	837	840	841.00	8
6:00 PM	20	839	841	842	840	840.50	3
6:30 PM	21	840	840	842	836	839.50	6
7:00 PM	22	844	845	841	843	843.25	4
7:30 PM	23	848	843	844	836	842.75	12
8:00 PM	24	840	844	841	845	842.50	5
8:30 PM	25	843	845	846	842	844.00	4
					Totals	**21036.25**	**129**

The data are arranged this way to help us determine whether the differences in the subgroups result from special causes over time. The measurement-to-measurement differences here are arranged to trap special variations over time between the subgroups and to confine the common process variation within the subgroups.

Using the subgroup range values and Equation (8.1),

$$\bar{R} = 129/25 = 5.16$$

From this, the control limits can be calculated using Equations (8.7) and (8.8). Using a subgroup size of 4, Table B.1 in Appendix B gives us values for D_3 = 0.00 and D_4 = 2.282. Hence,

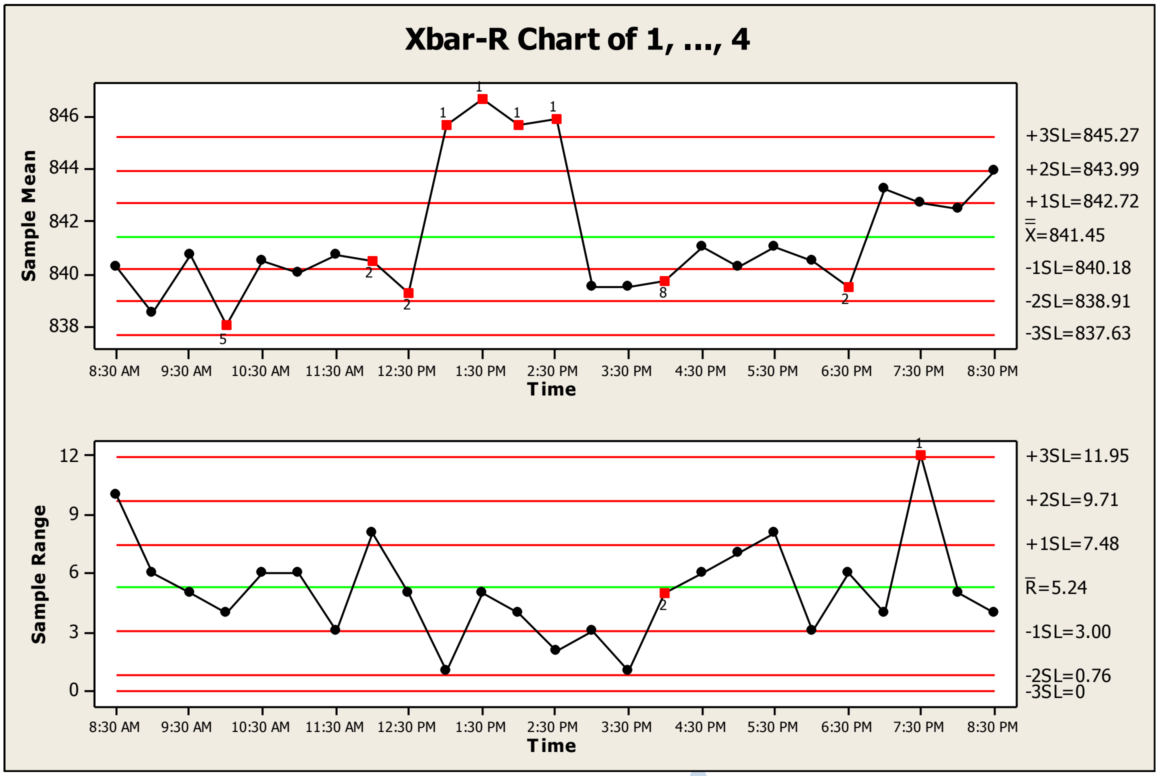
$$UCL(R) = (2.282)(5.16) = 11.78$$
$$LCL(R) = 0(5.16) = 0.00$$

Zone boundaries are computed using Equations (8.11) through (8.14):

Boundary between lower zones A and B = 5.16[1 - 2(0.880)/2.059] = 0.75
Boundary between lower zones B and C = 5.16[1 - 1(0.880)/2.059) = 2.95
Boundary between upper zones B and C = 5.16[1 + 1(0.880)/2.059] = 8.37
Boundary between upper zones A and B = 5.16[1 + 2(0.880)/2.059] = 9.57

The lower panel of Figure 8.4 illustrates the control chart for the range. Recall from Chapter 6 that a search for indications of a lack of control should always be made from right to left on the control chart; that is, the search should be made by looking backward in time from the present. Reading from right to left, the range at subgroup number 23 is beyond the upper control limit. Furthermore, subgroup number 16 is the eighth consecutive point below the centerline and therefore indicates a lack of control by virtue of the fourth rule presented in Chapter 6. Hence, there are two indications of a lack of control.

Figure 8.4
Initial $\bar{x}$ and R Chart for Tuning Knob Diameters



An investigation reveals that at 7:25 pm a water pipe burst in the lunchroom. The episode was not serious, but caused water to leak from the lunchroom onto the floor beneath the machinery involved in the process. This disruption seems to have caused the lack of control observed at subgroup 23. The operators believe this to be a special cause of variation that should not recur once the plumbing has been repaired. The initial study does not reveal any special source of variation for the indication of a lack of control at subgroup 16.

The data for subgroup 23 are then removed from the data set. The repair of the plumbing has permanently removed the conditions leading to this observation. The data for subgroup 16 are left in place, as no special cause of variation can be isolated and removed that would explain its presence. $\overline{R}$ is recomputed using Equation (8.1) to reflect the deletion of subgroup 23:

$$\overline{R} = 117/24 = 4.88$$

The revised control limits and zone boundaries are computed using Equations (8.7) and (8.8),and Equations (8.11) through (8.14):

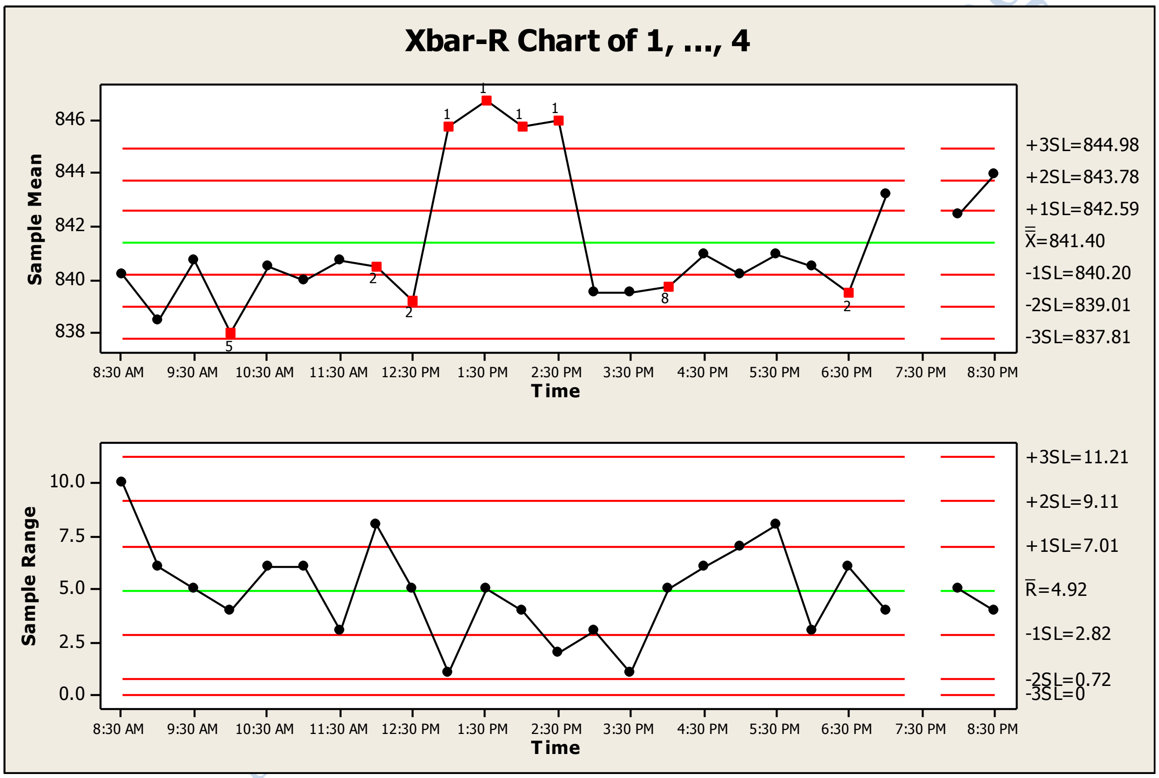
$$UCL(R) = (2.282)(4.88) = 11.14$$
$$LCL(R) = (0)(4.88) = 0.00$$

Boundary between lower zones A and B = 4.88[1 - 2(0.880)/2.059] = 0.71
Boundary between lower zones B and C = 4.88(1 - 1(0.880)/2.059) = 2.79

Boundary between upper zones B and C = 4.88(1+1(0.880)/2.059) = 6.97
Boundary between upper zones A and B = 4.88[1+ 2(0.880)/2.059] = 9.05

The revised R chart appears in the lower portion of Figure 8.5; because the centerline has been shifted, subgroup number 16 is no longer the eighth consecutive point below the centerline. There are no other indications of a lack of control in the data. Changing the centerline, control limits, and zone boundaries may uncover other indications of a lack of control, but this is not the case here. Hence, the $\bar{x}$ portion of the chart may be constructed using the average range from the now-stable R portion of the chart.

Figure 8.5
$\bar{x}$ and R Chart for Tuning Knob Diameters



The centerline of the $\bar{x}$ chart is the average of the subgroup averages, where $\bar{x}$ for subgroup 23 has been subtracted from the total in Table 8.2. Equation (8.16) yields

$$\bar{\bar{x}} = 20{,}193.50/24 = 841.40$$

The control limits and the zone boundaries can now be computed using Equations (8.19) and (8.21), and Equations (8.22) through (8.25):

$$UCL(\bar{x}) = 841.40 + (0.729)(4.88) = 844.96$$
$$LCL(\bar{x}) = 841.40 - (0.729)(4.88) = 837.84$$

Boundary between lower zones A and B = 841.40 - (2/3)(0.729)(4.88) = 839.03
Boundary between lower zones B and C = 841.40 - (1/3)(0.729)(4.88) = 840.21
Boundary between upper zones B and C = 841.40+ (1/3)(0.729)(4.88) = 842.59
Boundary between upper zones A and B = 841.40+ (2/3)(0.729)(4.88) = 843.77

The $\bar{x}$ portion of the control chart has been completed using these values and appears in the upper portion of Figure 8.5. Reading from right to left, there are several indications of a lack of control. Four points are beyond the upper control limit. These occurred sequentially from 1:00 pm to 2:30 pm. At 10:00 am (fourth subgroup), the average is 838.00, the second of three consecutive points in the lower zone A. This indicates a lack of control by virtue of Rule 2 of Chapter 6. The 8^{th} and 9^{th} subgroups form a run of 8 or more subgroups below the centerline which indicates a possible out of control situation. The 16^{th} subgroup forms a run of 8 subgroups in a row more than 1 standard deviation from the mean. The 21^{st} subgroup is the 8^{th} subgroup below the centerline which indicates another potential lack of control.

There is very little doubt that there is at least one source of special variation acting on this process. Finding that source requires further investigation, which leads to the discovery that at 12:50 pm (just after the selection of subgroup number 9), a keyway wedge had cracked and needed to be replaced on the machine. The mechanic who normally makes this repair was out on a repair call between 12:45 and 2:45, so the machine operator made the repair. This individual had not been properly trained for the repair, so the wedge was not properly aligned in the keyway and subsequent points were out of control. Both the operator and the mechanic agree that the need for this repair was not unusual. To correct this problem, management and labor agree to train the machine operator and provide the appropriate tools for making this repair in the mechanic's absence. Furthermore, the maintenance and engineering staffs agree to search for a replacement part for the wedge that is less prone to cracking.
All other sources of possible special causes could be artifacts of the out-of-control situation caused by subgroups 10 through 13; once these subgroups are dropped and the control chart statistics are recalculated, the process may be in statistical control. Accordingly, we recompute the values for the relevant statistics:

$$\bar{\bar{x}} = 16{,}809.25/20 = 840.46$$
$$\bar{R} = 105/20 = 5.25$$

Using these, the control limits and zone boundaries for the revised R chart become

UCL(R) = (2.282)(5.25) = 11.98
LCL(R) = 0(5.25) = 0.00

Boundary between lower zones A and B = 5.25[1 - 2(0.880)/2.059] = 0.76
Boundary between lower zones B and C = 5.25[1 - 1(0.880)/2.059] = 3.01
Boundary between upper zones B and C = 5.25[1+ 1(0.880)/2.059] = 7.49
Boundary between upper zones A and B = 5.25[1+ 2(0.880)/2.059] = 9.74

The control chart for the range portion is shown in the lower portion of Figure 8.6. Recall that although the previously out-of-control value for point 23 is still on the Minitab control chart, it is no longer in the data set. There are no indications of a lack of control on the R chart, so the $\bar{x}$ chart may be constructed. The control limits and zone boundaries are:

$$UCL(\bar{x}) = 840.46 + (0.729)(5.25) = 844.29$$
$$LCL(\bar{x}) = 840.46 - (0.729)(5.25) = 836.63$$

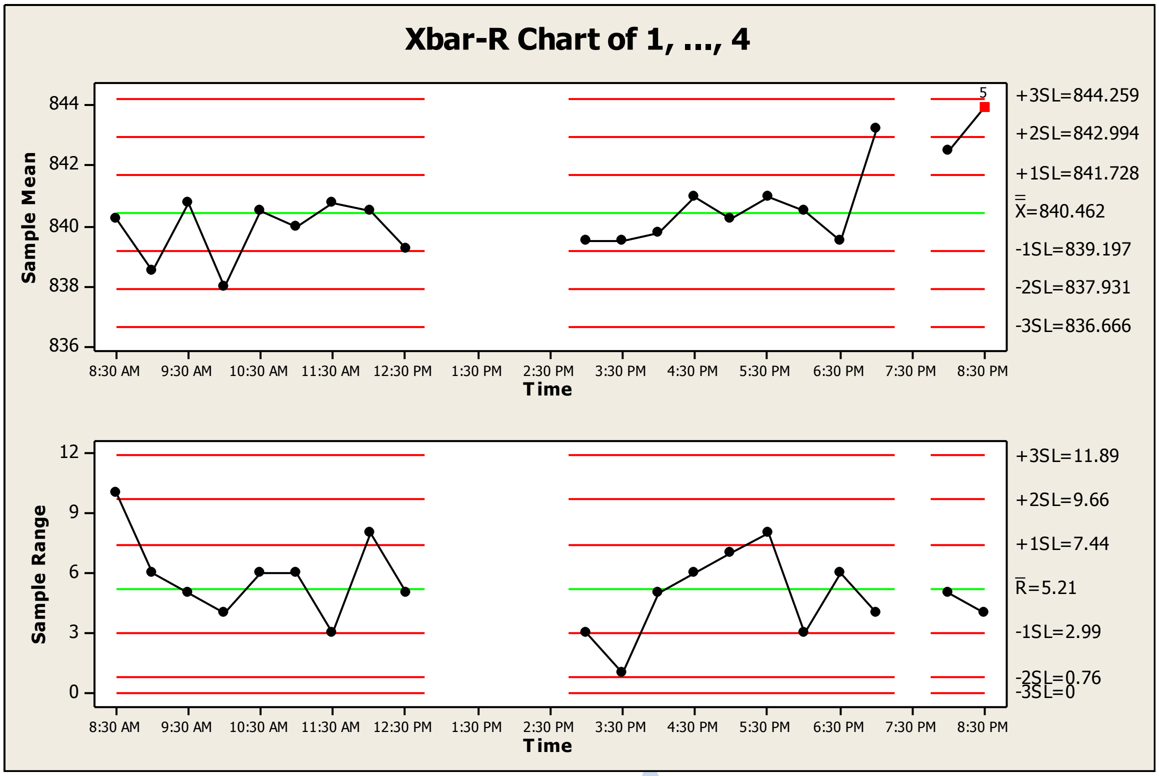
Boundary between lower zones A and B = 840.46 - (2/3)(0.729)(5.25) = 837.91
Boundary between lower zones B and C = 840.46 - (1/3)(0.729)(5.25) = 839.18
Boundary between upper zones B and C = 840.46+ (1/3)(0.729)(5.25) = 841.74
Boundary between upper zones A and B = 840.46+ (2/3)(0.729)(5.25) = 843.01

The $\bar{x}$ portion of the control chart is shown in the upper portion of Figure 8.6. The last data point, number 25, now indicates a lack of control. That is, if we ignore the now-eliminated entry at data point number 23, data point number 25 is the second of three consecutive points in zone A or beyond, indicating a lack of control by virtue of the second rule of Chapter 6.

At times a conservative approach is warranted -- that is, detection of patterns indicating a lack of control will usually be followed by a somewhat costly search for the special cause(s) of variation. A process that has just been altered to eliminate one or more special sources of variation may be allowed to run for a while longer to determine whether any special sources of variation are really present or whether the indication of a lack of control is only a temporary effect that can be attributed to the removal of some of the data. If special sources of variation are present, the data taken from the process output will soon demonstrate evidence of that variation.

This process can be permitted to run as if it were statistically controlled. However, it must be watched closely to ensure that the special sources of variation have been removed and are no longer affecting the process.

Figure 8.6
Revised $\bar{x}$ and R Chart for Tuning Knob Diameters



8.5 $\bar{x}$ and s Charts

$\bar{x}$ and s charts are quite similar to $\bar{x}$ and R charts, providing the same sort of information -- but $\bar{x}$ and s charts are used when subgroups consist of 10 or more observations.

In Chapter 5 we saw that the standard deviation of the process output, σ, could be estimated using s, which is computed from Equation (5.10). s provides an estimate that generally has a smaller standard error than R. The benefits of having a smaller standard error must be weighed against the costs of larger subgroup sizes and more complex calculations.

The $\bar{x}$ and s chart is used with larger subgroup sizes, and larger subgroups are not always desirable. One reason for this is that s may be viewed as a less robust estimator of the population standard deviation than R, since R is more sensitive to shifts in population shape than s. For subgroup sizes of fewer than 10, the range provides a reasonable statistic with which to estimate the standard error. Also, the range is easier to calculate than s, which gives it an advantage in many situations. So when subgroup sizes are small, the range is used as an estimator for σ.

When subgroup sizes are 10 or more, s is almost always used because as subgroup size increases, s becomes a much more statistically efficient estimator for σ. When the subgroup size is increased, the likelihood of encountering an extreme value increases, so that s, which is less affected than R by extreme values in the data, becomes a better estimator for σ.

Historically, subgroup ranges have been preferred because they are easier to compute than subgroup standard deviations. As the use of software has grown, the need to avoid tedious computations has decreased, and the reluctance to use $\bar{x}$ and s charts has decreased.

8.5.1 The s Portion

The construction of the $\bar{x}$ and s chart parallels that of the $\bar{x}$ and R chart. Both charts begin with an examination of the portion of the chart concerned with the variability of the process. The standard deviation, s, must be calculated for each subgroup. The value for s is the basis for an estimate of the process standard deviation, from which a set of factors for the control limits is developed.

Equation (5.10) is used to compute s for each subgroup:

$$s = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n-1}}$$

where n is the number of observations in each subgroup (subgroup size). The sequence of s values is then averaged, yielding $\bar{s}$, the centerline for the s chart:

$$\text{Centerline (s)} = \bar{s} = \Sigma\, s/k \qquad (8.26)$$

where k is the number of subgroups.

$\bar{s}$ is used to form an estimate of the process standard deviation, σ:

$$\sigma = \bar{s}/c_4 \qquad (8.27)$$

where c_4 is a factor that depends on the subgroup size, assumes that the process characteristic is stable and normally distributed, and is given in Table B.1 in Appendix B.

Control limits for the s chart are constructed by adding and subtracting three times the standard error of s from the centerline of the control chart:

$$\text{UCL(s)} = \bar{s} + 3\sigma_{\sigma} \qquad (8.28)$$

$$\text{LCL(s)} = \bar{s} - 3\sigma_{\sigma} \qquad (8.29)$$

Assuming that the distribution of the process output was and will be stable, we can derive control limits. (As indicated in Chapter 6, the assumption of normality is not necessary to interpret the control limits; we can use the Empirical Rule instead.)

The sampling distribution of the standard deviation of a stable process has a standard error given by

$$\sigma_\sigma = \sigma\sqrt{1-c_4^2} \tag{8.30}$$

Hence, the upper control limit for the s chart is

$$\text{UCL}(s) = \bar{s} + 3\sigma\sqrt{1-c_4^2} \tag{8.31}$$

Using $\bar{s}/c_4$ to estimate σ yields

$$\text{UCL}(s) = \bar{s} + \frac{3\bar{s}\sqrt{1-c_4^2}}{c_4} = \bar{s}\left[1 + \frac{3\sqrt{1-c_4^2}}{c_4}\right] \tag{8.32}$$

Similarly,

$$\text{LCL}(s) = \bar{s} - \frac{3\bar{s}\sqrt{1-c_4^2}}{c_4} = \bar{s}\left[1 - \frac{3\sqrt{1-c_4^2}}{c_4}\right] \tag{8.33}$$

A more convenient way to represent these control limits is by defining two new constants, B_3 and B_4:

$$B_3 = 1 - \frac{3\sqrt{1-c_4^2}}{c_4} \tag{8.34}$$

$$B_4 = 1 + \frac{3\sqrt{1-c_4^2}}{c_4} \tag{8.35}$$

so that

$$\text{UCL}(s) = B_4\bar{s} \tag{8.36}$$

$$\text{LCL}(s) = B_3\bar{s} \tag{8.37}$$

Values for B_3 and B_4 depend on subgroup size and can be found in Table B.1 in Appendix B.

Boundaries for zones A, B, and C for the s chart are placed at the usual multiples of one and two times the standard error on either side of the centerline. As negative values are meaningless, the zone boundaries cease to exist below the 0.00 line on the control chart. To find the zone boundaries, we divide the difference between the upper control limit and the centerline by 3. This value provides an estimate for the standard error of $\bar{s}$.

Adding and subtracting this value from the centerline yields the upper and lower boundaries between zones B and C, respectively, while adding and subtracting two times this value yields the upper and lower boundaries between zones A and B, respectively. These boundaries can be expressed as

Boundary between lower zones A and B = $\bar{s}$ - (2/3) $\bar{s}$ (B_4 - 1) (8.38)

Boundary between lower zones B and C = $\bar{s}$ - (1/3) $\bar{s}$ (B_4 - 1) (8.39)

Boundary between upper zones B and C = $\bar{s}$ + (1/3) $\bar{s}$ (B_4 - 1) (8.40)

Boundary between upper zones A and B = $\bar{s}$ + (2/3) $\bar{s}$ (B_4 - 1) (8.41)

If the s portion of the control chart is found to be stable, the $\bar{x}$ portion may be constructed. However, if the s portion indicates of a lack of statistical control, then the $\bar{x}$ portion cannot be safely evaluated until any special sources of variation have been removed and the process stabilized. As we will see, this is because the estimate of the standard error of the x-bar portion is based on the average value of s. Just as with the $\bar{x}$ and R chart, if the variability is not in control, then estimates of the standard error are unreliable, leading to unreliable control limits for $\bar{x}$.

8.5.2 The $\bar{x}$ Portion

The centerline for the $\bar{x}$ chart is the average of the subgroup averages, $\bar{\bar{x}}$, and can be found using Equation (8.16). The control limits are found by adding and subtracting three times the standard error of $\bar{X}$ from the centerline:

$$\bar{\bar{x}} \pm 3\sigma/\sqrt{n} \qquad (8.42)$$

Our estimate of the process standard deviation is, from Equation (8.27):

$$\sigma = \bar{s}/c_4$$

so that the control limits become

$$\bar{\bar{x}} \pm 3(\bar{s}/c_4)/\sqrt{n} = \bar{\bar{x}} \pm 3\bar{s}/(c_4\sqrt{n}) \qquad (8.43)$$

Letting the constant $A_3 = 3/(c_4\sqrt{n})$, the control limits for the $\bar{x}$ portion can be expressed as:

$$UCL(\bar{x}) = \bar{\bar{x}} + A_3\bar{s} \qquad (8.44)$$

and

$$LCL(\bar{x}) = \bar{\bar{x}} - A_3\bar{s} \qquad (8.45)$$

Where the value for A_3 depends on subgroup size and can be found in Table B.1 in Appendix B.

The zone boundaries are placed at one and two times the standard error on either side of the centerline. The standard error is $\bar{s}/(c_4\sqrt{n})$, so that the zone boundaries are:

Boundary between lower zones A and B = $\bar{\bar{x}} - 2\bar{s}/(c_4\sqrt{n})$ (8.46)

Boundary between lower zones B and C = $\bar{\bar{x}} - \bar{s}/(c_4\sqrt{n})$ (8.47)

Boundary between upper zones B and C = $\bar{\bar{x}} + \bar{s}/(c_4\sqrt{n})$ (8.48)

Boundary between upper zones A and B = $\bar{\bar{x}} + 2\bar{s}/(c_4\sqrt{n})$ (8.49)

8.5.3 $\bar{x}$ and s Charts: An Example

In a converting operation, a plastic film is combined with paper coming off a spooled reel. As the two come together, they form a moving sheet that passes as a web over a series of rollers. The operation runs in a continuous feed, and the thickness of the plastic coating is an important product characteristic. Coating thickness is monitored by a highly automated piece of equipment that uses 10 heads to take 10 measurements across the web at half-hour intervals. Table 8.4 shows a sequence of measurements taken over 20 time periods.

Table 8.4

Plastic Coating Thickness

Head Number	8:30	9:00	9:30	10:00	10:30	11:00	11:30
1	2.08	2.14	2.30	2.01	2.06	2.14	2.05
2	2.26	2.02	2.10	2.10	2.12	2.22	1.97
3	2.13	2.14	2.20	2.15	1.98	2.18	2.05
4	1.94	1.94	2.25	1.97	2.12	2.27	2.16
5	2.30	2.30	2.05	2.25	2.20	2.17	2.02
6	2.15	2.08	1.95	2.12	2.02	2.26	2.02
7	2.07	1.94	2.10	2.10	2.19	2.15	2.14
8	2.02	2.12	2.16	1.90	2.03	2.07	2.07
9	2.22	2.15	2.37	2.04	2.02	2.02	2.00
10	2.18	2.36	1.98	2.08	2.09	2.36	2.05
$\bar{x}$	**2.14**	**2.12**	**2.15**	**2.07**	**2.08**	**2.18**	**20.6**
s	**0.111**	**0.137**	**0.136**	**0.980**	**0.740**	**0.099**	**0.059**
Head Number	**12:00**	**12:30**	**13:00**	**13:30**	**14:00**	**14:30**	**15:00**
1	2.08	2.13	2.13	2.24	2.25	2.03	2.08
2	2.31	1.90	2.16	2.34	1.91	2.10	1.92

3	2.12	2.12	2.12	2.40	1.96	2.24	2.14
4	2.18	2.04	2.22	2.26	2.04	2.20	2.20
5	2.15	2.40	2.12	2.13	1.93	2.25	2.02
6	2.17	2.12	2.07	2.15	2.08	2.03	2.04
7	1.98	2.15	2.04	2.08	2.29	2.06	1.94
8	2.05	2.01	2.28	2.02	2.42	2.19	2.05
9	2.00	2.30	2.12	2.05	2.10	2.13	2.12
10	2.26	2.14	2.10	2.18	2.00	2.20	2.06
$\bar{x}$	**2.13**	**2.13**	**2.14**	**2.19**	**2.10**	**2.14**	**2.06**
s	**0.107**	**0.141**	**0.070**	**.0125**	**0.170**	**0.084**	**0.086**
Head Number	**15:30**	**16:00**	**16:30**	**17:00**	**17:30**	**18:00**	
1	2.04	1.92	2.12	1.98	2.08	2.22	
2	2.14	2.10	2.30	2.30	2.12	2.05	
3	2.18	2.13	2.01	2.31	2.11	1.93	
4	2.12	2.02	2.20	2.12	2.22	2.08	
5	2.00	1.93	2.11	2.08	2.00	2.15	
6	2.02	2.17	1.93	2.10	1.95	2.27	
7	2.05	2.24	2.02	2.15	2.15	1.95	
8	2.34	1.98	2.25	2.35	2.14	2.11	
9	2.12	2.34	2.05	2.12	2.28	2.12	
10	2.05	2.12	2.10	2.26	2.31	2.10	
$\bar{x}$	**2.11**	**2.10**	**2.11**	**2.18**	**2.14**	**2.10**	
s	**0.101**	**0.136**	**0.115**	**0.121**	**0.113**	**0.106**	

Arranging the data in this way helps us determine whether there are any special causes of variation from subgroup to subgroup, over time. Values for $\bar{x}$ and s, the subgroup means and standard deviations, have been computed for each subgroup. The values for the averages of these subgroup means and standard deviations are computed using Equations (8.16) and (8.26):

$$\bar{\bar{x}} = 42.43/20 = 2.12$$

and

$$\bar{s} = 2.19/20 = 0.11$$

The s chart must be constructed first, and Equations (8.36) and (8.37) provide the upper and lower control limits:

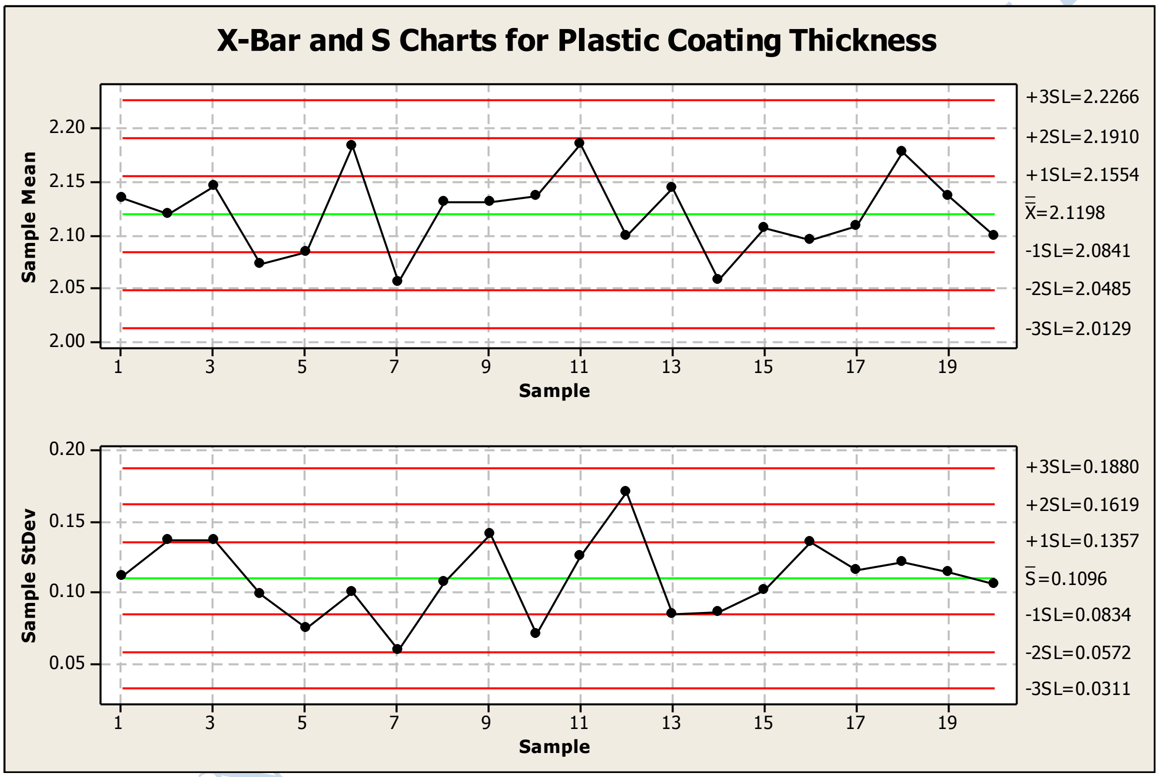
UCL(s) = (1.716)(0.11) = 0.189
LCL(s) = (0.284)(0.11) = 0.031

The zone boundaries are computed using Equations (8.38) through (8.41):

Boundary between lower zones A and B= 0.11 - (2/3)(0.11)(1.716 - 1)= 0.057
Boundary between lower zones B and C= 0.11 - (1/3)(0.11)(1.716 - 1)= 0.084
Boundary between upper zones B and C= 0.11+ (1/3)(0.11)(1.716 - 1)= 0.136
Boundary between upper zones A and B= 0.11+ (2/3)(0.11)(1.716 - 1)= 0.163

The control chart for the standard deviation is shown in the lower portion of Figure 8.7. As the control chart does not indicate a lack of control, the process variability appears stable. The $\bar{x}$ portion of the chart may now be constructed.

Figure 8.7
$\bar{x}$ and s Chart for Plastic Coating Thickness



It would not have been correct to analyze the $\bar{x}$ portion before establishing the stability of the process standard deviation. If the standard deviation were not stable, the mean value of the subgroup standard deviations would be unreliable; the control limits on the $\bar{x}$ chart would also be unreliable, as the control limits are based on the value for s. The control chart might then fail to reveal patterns indicating a lack of control when they existed. This is why the estimate of the process standard deviation is not based on the standard deviation taken across all observations.

The average value for x, $\bar{x}$, is 2.12. Equations (8.44) and (8.45) are used to form the upper and lower control limits for the $\bar{x}$ chart:

$$UCL(\bar{x}) = 2.12 + (0.975)(0.11) = 2.227$$
$$LCL(\bar{x}) = 2.12 - (0.975)(0.11) = 2.013$$

We then use Equations (8.46) through (8.49) to calculate the zone boundaries:

Boundary between lower zones A and B = 2.12 - 2{0.11/[(0.9727)(3.16)]}= 2.048
Boundary between lower zones B and C = 2.12 - 1{0.11/[(0.9727)(3.16)]}= 2.084
Boundary between upper zones B and C= 2.12 + 1{0.11/[(0.9727)(3.16)]}= 2.156
Boundary between upper zones A and B= 2.12 + 2{0.11/[(0.9727)(3.16)]}= 2.192

The $\bar{x}$ chart appears in the upper portion of Figure 8.7 and completes the control chart. There are no indications of a lack of control, so the process can be considered to be stable and the output predictable with respect to time, as long as conditions remain the same.

It is now appropriate to use some of the methods that will be described in Chapter 10 (such as check sheets, Pareto analysis, or brainstorming) to attempt to reduce the common causes of variation in the never-ending quest to decrease the difference between process performance and customer needs.

8.6 Individuals and Moving Range Charts

It is not uncommon to encounter a situation where only one measurement or data point is observed per subgroup. Perhaps such measurements are monthly revenues, quarterly costs, cycle time per customer, waiting time per patient; or the measurements are destructive and/or expensive; or perhaps they represent a single batch where only one measurement is appropriate, such as the total yield of a homogeneous chemical batch process. Whatever the case, there are circumstances when data must be taken as individual units.

Individuals and moving range charts have two parts: one charting the process variability and the other charting the process average for the single measurements. The two parts are used in tandem much as in the $\bar{x}$ and R chart. Stability must first be established in the portion charting the variability, because the estimate of the process variability provides the basis for the control limits of the portion charting the process average.

Single measurements of variables are considered a subgroup of size one. Hence, there is no variability within the subgroups themselves, and an estimate of the process variability must be made in some other way. An estimate of variability is based on the point-to-point variation in the sequence of single values, measured by the moving range (the absolute value of the difference between each data point and the one that immediately preceded it):

$$R = |x_i - x_{i-1}| \qquad (8.50)$$

An average of the moving ranges is used as the centerline for the moving range portion of the chart and as the basis of an estimate of the overall process variation:

$$\text{Centerline (moving range)} = \overline{R} = \Sigma\ R/(k - 1) \quad (8.51)$$

where k is the number of single measurements. As it is impossible to calculate the moving range for the first subgroup because none precedes it, there are only k - 1 range measurements; thus the sum of the R values is divided by k - 1.

From Equations (8.3) and (8.4), subgroup ranges have an average given by $\overline{R} = d_2\sigma$, and a standard error of $\sigma_R = d_3\sigma$, where σ is the process standard deviation. The estimate of the process variation is subsequently used to create the 3-sigma control limits for both the moving range portion and the individuals portion of the control chart. As we saw earlier in this chapter, the control limits for a range are given by:

$$\text{UCL (moving range)} = D_4\overline{R} \quad (8.52)$$

$$\text{LCL (moving range)} = D_3\overline{R} \quad (8.53)$$

where D_3 and D_4 depend on subgroup size and are given in Table B.1 in Appendix B. In this case D_3 is 0.000 and D_4 is 3.267 because the subgroup size for the moving range portion is 2.

For the individuals portion of the control chart, the centerline is the average of the single measurements. We find the control limits by adding and subtracting three times the standard deviation of the single measurements, estimated by $\overline{R}/d_2$:

$$\text{Centerline (x)} = \overline{x} = \Sigma\, x/k \quad (8.54)$$

$$\text{UCL (x)} = \overline{x} + 3\left(\overline{R}/d_2\right) \quad (8.55)$$

Using the factor E_2 to represent $3/d_2$, the expression for the upper control limit becomes

$$\text{UCL (x)} = \overline{x} + E_2\overline{R} \quad (8.56)$$

where E_2 depends on subgroup size and can be found in Table B.1 in Appendix B. In this case the subgroup size is 2, as we use two observations to calculate each moving range value. Hence, $E_2 = 2.66$, and

$$\text{UCL (x)} = \overline{x} + 2.66\ \overline{R} \quad (8.57)$$

Similarly, the lower control limit is found using:

$$\text{LCL (x)} = \overline{x} - 2.66\ \overline{R} \quad (8.58)$$

8.6.1 An Example

A chemical company produces 2,000-gallon batches of a liquid chemical product, A-744, once every two days. The product is a combination of six raw materials, of which three are liquids and three are powdered solids. Production takes place in a single tank, agitated as the ingredients are added, and for several hours thereafter. Shipments of A-744 to the customer are made in bins as single lots when the batches are finished. The chemical company is concerned with the density of the finished product, which it measures in grams per cubic centimeter. As batches are constantly stirred during production, the density is assumed to be relatively uniform throughout each batch. Therefore, management decides that density will be measured by only one reading per batch. During a 60-day period, 30 batches of A-744 are produced. Table 8.5 shows the density readings for these batches.

Table 8.5

A-744 Batch Density

Row	Date	Density	Row	Date	Density	Row	Date	Density
1	5/6	1.242	11	5/29	1.299	21	6/21	1.281
2	5/8	1.289	12	5/31	1.225	22	6/24	1.274
3	5/10	1.186	13	6/3	1.185	23	6/26	1.234
4	5/13	1.197	14	6/5	1.194	24	6/28	1.187
5	5/15	1.252	15	6/7	1.235	25	7/1	1.196
6	5/17	1.221	16	6/10	1.253	26	7/3	1.282
7	5/20	1.299	17	6/12	1.329	27	7/5	1.322
8	5/22	1.234	18	6/14	1.275	28	7/8	1.258
9	5/24	1.288	19	6/17	1.232	29	7/9	1.261
10	5/27	1.193	20	6/19	1.201	30	7/11	1.201

Using Equation (8.50), we calculate the moving range by subtracting the previous observation from the next observation and then taking the absolute value. For example, the first moving range is obtained by subtracting the first observation X_1 = 1.242 from the second observation X_2 = 1.289. Thus, the first moving range is 1.289 – 1.242 = 0.047. This process continues until the next to last observation X_{29} = 1.261 is subtracted from the last observation X_{30} = 1.201, or a moving range of 0.60. Since there is no moving range for the first observation, there is one fewer moving range than there are observations.

Using Equation (8.51), the average of the 29 moving range values is

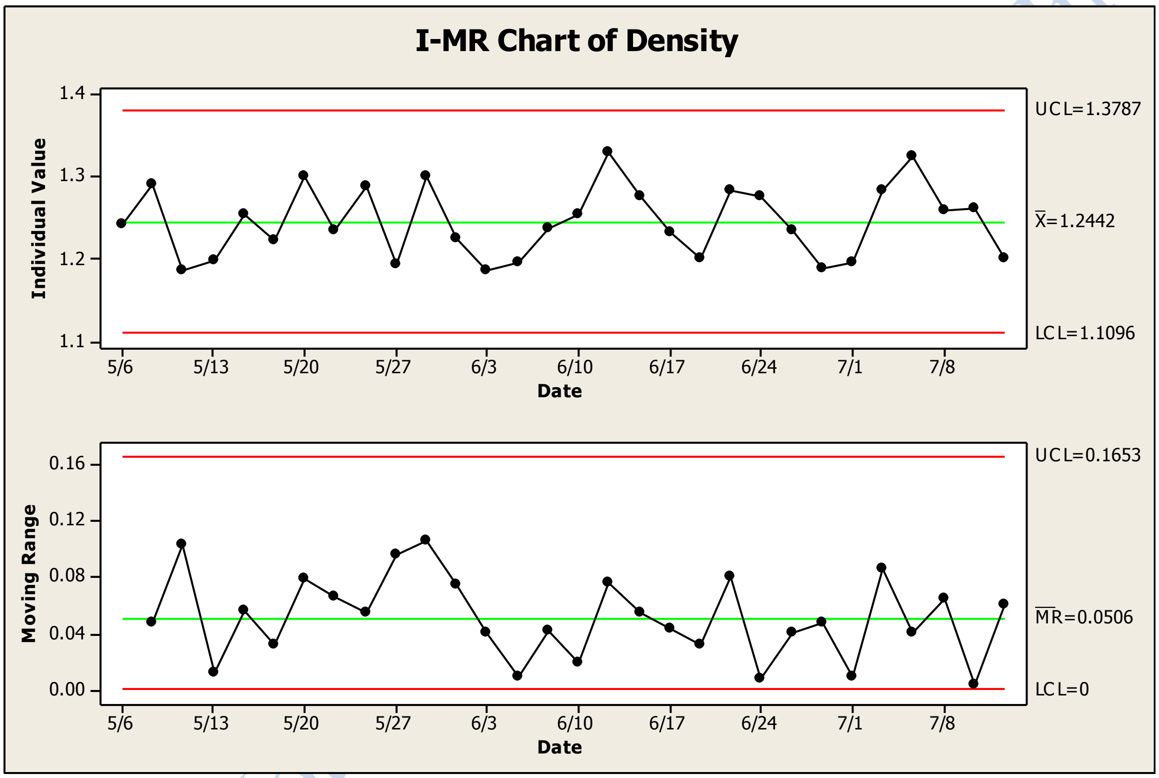
$$\text{Centerline (moving range)} = \bar{R} = 0.0506$$

The control limits for the moving range portion of the control chart can be found using Equations (8.52) and (8.53):

$$\text{UCL (moving range)} = D_4\,\overline{R} = (3.267)(0.0506) = 0.1653$$
$$\text{LCL (moving range)} = D_3\,\overline{R} = (0.000)(0.0506) = 0.000$$

The control chart for the moving ranges is shown in the lower portion of Figure 8.8. The moving range appears to be in a state of statistical control, so it is safe to use the average moving range value to construct the single measurements portion of the chart.

Figure 8.8
Individuals and Moving Range Chart for A-744 Density



Using Equation (8.54), the centerline for the individuals control chart is:

$$\text{Centerline (x)} = \overline{x} = 1.2442$$

The control limits for the individuals portion are found using Equations (8.57) and (8.58):

$$\text{UCL (x)} = \overline{x} + 2.66\,\overline{R} = 1.250 + 2.66\ (0.0506) = 1.3787$$
$$\text{LCL (x)} = \overline{x} - 2.66\ \overline{R} = 1.250 - 2.66\ (0.0506) = 1.1096$$

The upper portion of Figure 8.8 illustrates the control chart for the individuals portion. The process appears to be in a state of statistical control, since there are no points beyond the control limits and no other signs of any trends or patterns in the data.

8.6.2 Special Characteristics of Individuals and Moving Range Charts

Individuals and moving range charts have certain unique characteristics that distinguish them from other control charts because each subgroup consists of only one value, and consequently, process variation is estimated on the basis of observation-to-observation changes For example, for an individuals and moving range chart to be reliable, it is best to have at least 100 subgroups, whereas 25 will suffice for most other control chart forms.

Correlation in the Moving Range. The moving ranges tend to be correlated. For example, a data point near the centerline followed by one in the upper zone A, followed by one in the lower zone C, will result in two large successive moving range points. As a consequence, large moving range values tend to be followed by other large moving range values, and small moving range values tend to be followed by other small moving range values. Because of this, users must be cautious in applying rules for a lack of control dealing with patterns in the data. For example, a false alarm may be created by using the rule designating the second of two out of three consecutive points in zone A or beyond as indicating a lack of control. For this reason, it is usually best to be conservative when applying the rules concerning patterns in the data other than points beyond the control limits that indicate a lack of control in moving range charts. For example, instead of 8 consecutive values above or below the centerline indicating a lack of control, we might require 10 or 12. Knowledge and experience are the best guides in establishing policy in this case.

Inflated Control Limits. The control limits for individuals and moving range charts are computed from individual measurements, not from statistics (e.g., mean, range or standard deviation). The individual measurements will fall within control limits if the variation in the process over the long run is approximately the same as the variation in the short run. Because the process variation is estimated using the moving range, the short-run (between-consecutive-subgroups) variation in the process provides the estimate of the process variation.

Changes in either the short-run or long-run variation may produce unreliable control limits. One indication that the control limits are unreliable is the occurrence of at least two-thirds of the data points below the centerline of the moving range portion of the control chart. [Shewhart, p.314] When this happens, control limits should be based on the median of the moving range values, M_{e_R} , rather than on the average of the moving range values. M_{e_R} is calculated using the following procedure:

(1) Arrange the subgroup ranges from low to high.
(2) If there are an odd number of subgroup ranges, select the middle subgroup range as the **median range**.
(3) If there are an even number of subgroup ranges, select the two middle most subgroup ranges and compute their average. This average is the median range.

If **inflated control limits** are suspected, control limits based on the median of the moving ranges should be calculated and compared to those based on the average moving range; the narrower of the two sets should be used. Control limits for the moving range portion, based on the median, can be computed using

$$\text{Centerline (median moving range)} = M_{e_R} \qquad (8.59)$$

$$\text{UCL (median moving range)} = D_6 M_{e_R} \qquad (8.60)$$

$$\text{LCL (median moving range)} = D_5 M_{e_R} \qquad (8.61)$$

where values for D_5 and D_6 depend on subgroup size, assume stability and a normal distribution of the process characteristic, and can be found in Table B.1 in Appendix B. For the individuals chart the subgroup size is two, so the values used are $D_6 = 3.865$ and $D_5 = 0.000$. Hence,

$$\text{UCL (median moving range)} = 3.865\, M_{e_R} \qquad (8.62)$$

$$\text{LCL (median moving range)} = 0.000 \qquad (8.63)$$

(Recall that the assumption of normality is not required to interpret the control limits; the Empirical Rule may be used instead.)

The standard error of the single measurements is estimated using M_{e_R}/d_4, where values for d_4 depend on subgroup size and assume stability of the process characteristic. They can be found in Table B.1 in Appendix B. For subgroup size two, $d_4 = 0.954$. Hence, control limits for the single measurements portion are created by adding and subtracting three times $M_{e_R}/0.954$ from the centerline:

$$\text{UCL}(x) = \bar{x} + 3M_{e_R}/0.954 = \bar{x} + 3.145M_{e_R} \qquad (8.64)$$

Similarly,

$$\text{LCL}(x) = \bar{x} - 3.145M_{e_R} \qquad (8.65)$$

Consider the example of the manufacturer of chemicals discussed earlier. The chemical, A-744, is used by the manufacturer's customer as an ingredient in another process sensitive to the quantity of A-744. The manufacturer's customer is seeking to reduce costs by using the A-744 in whole bin lots. As the yield of the batches of A-744 can be expected to vary from its 2,000-gallon target, that yield is a likely candidate for the use of an Individuals control chart. The yields of the 30 batches of A-744 are each carefully measured, yielding a sequence of 30 single values. The data and the computed moving ranges are shown in Table 8.6.

Table 8.6

A-744 Batch Yields

Date	Yield	Moving	Date	Yield	Moving

		Range			Range
5/6	1989.0		6/10	2002.3	4.9
5/8	1998.9	9.9	6/12	1999.5	2.8
5/10	2027.4	28.5	6/14	2000.8	1.3
5/13	2001.5	25.9	6/17	2022.4	21.6
5/15	1991.3	10.2	6/19	1998.3	24.1
5/17	2001.3	10	6/21	1999.8	1.5
5/20	1997.4	3.9	6/24	2000.9	1.1
5/22	1989.3	8.1	6/26	1994.3	6.6
5/24	1995.5	6.2	6/28	1998.7	4.4
5/27	2014.4	18.9	7/1	2013.5	14.8
5/29	1990.2	24.2	7/3	1998.1	15.4
5/31	1999.6	9.4	7/5	2002.5	4.4
6/3	2008.1	8.5	7/8	2000.2	2.3
6/5	1999.4	8.7	7/9	1996.1	4.1
6/7	1997.4	2	7/11	2020.9	24.8
			Totals	**60049.0**	**308.5**

The centerline and control limits for the moving range portion of the control chart are found using Equations (8.51) through (8.53):

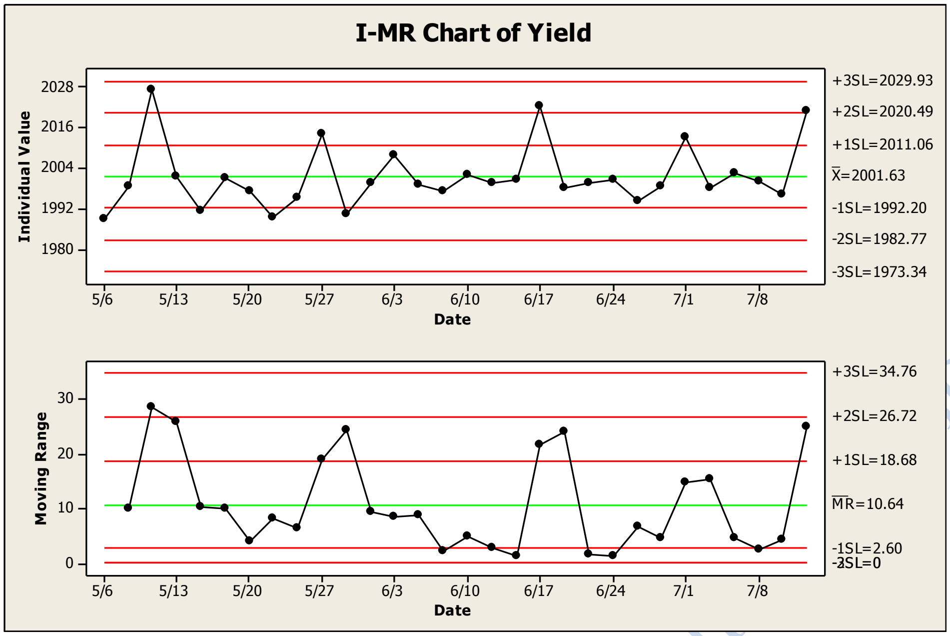
$$\overline{R} = 308.5/29 = 10.64$$

$$\text{UCL (moving range)} = D_4\overline{R} = (3.267)(10.64) = 34.76$$

$$\text{LCL (moving range)} = 0.00$$

The control chart for the moving range is shown in the lower portion of Figure 8.9. The moving range appears stable, so the average moving range value can be used to construct the individuals portion of the chart.

Figure 8.9
Individuals and Moving Range chart for A-744 Batch Yields



Using Equation (8.54), the average of the 30 yields is:

$$\bar{x} = 60{,}049.0/30 = 2001.63$$

The control limits for the individuals portion are found using Equations (8.57) and (8.58):

$$UCL\ (x) = \bar{x} + 2.66\ \bar{R} = 2001.63 + (2.66)(10.64) = 2029.93$$
$$LCL\ (x) = \bar{x} - 2.66\ \bar{R} = 2001.63 - (2.66)(10.64) = 1973.33$$

The control chart is shown in the upper portion of Figure 8.9. This portion also appears to be in a state of statistical control. However, an experienced eye detects that more than two-thirds of the data points (20 of the 29 moving ranges) are below the centerline, indicating that the control limits may be artificially inflated and therefore may be hiding indications of special sources of variation.

The median of the 29 moving range values is 8.5. This value can be used to calculate an alternate centerline and set of control limits using Equations (8.59) through (8.61) for the moving range portion of the control chart:

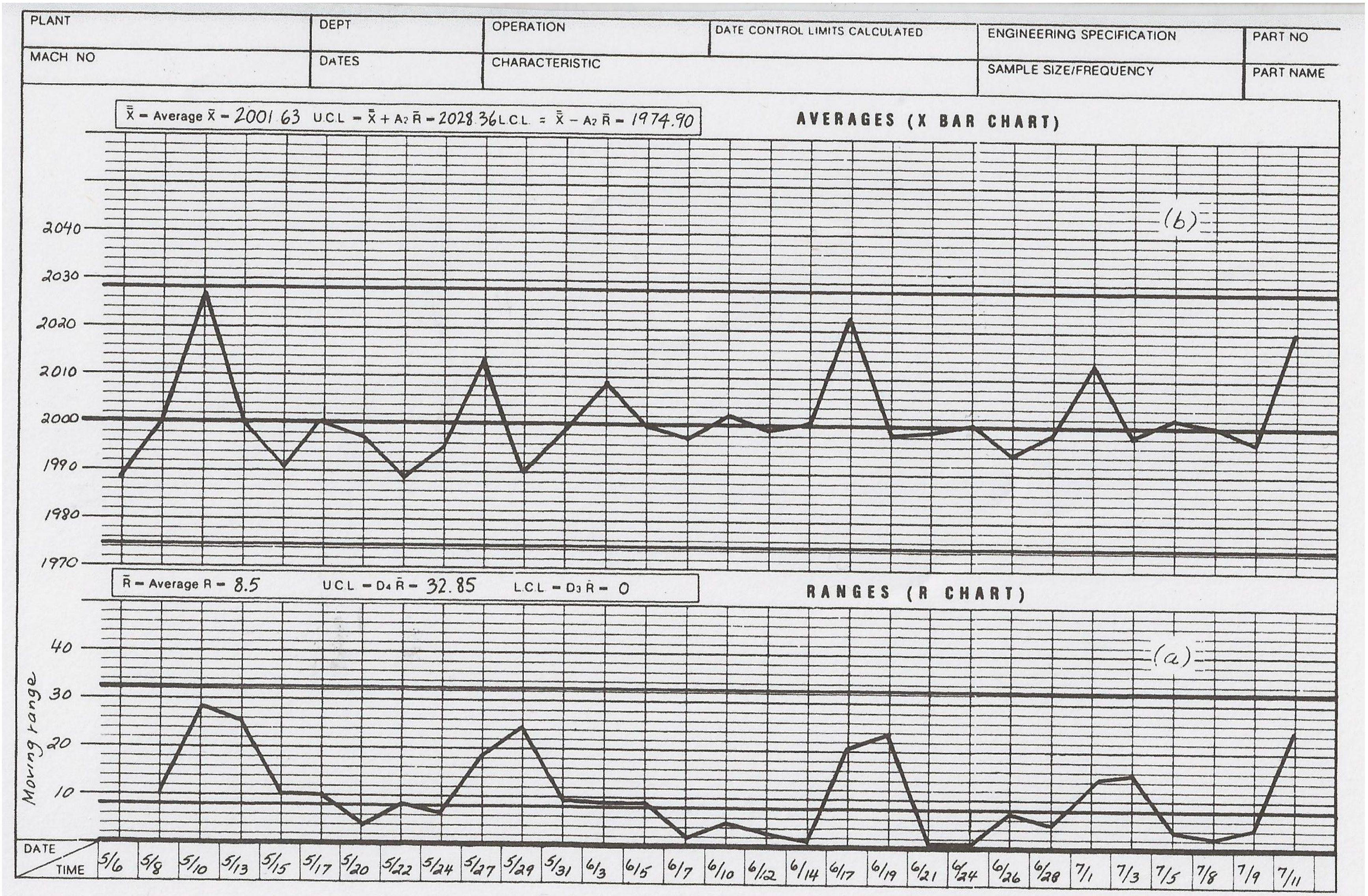
Centerline (median moving range) = 8.5

UCL (median moving range) = (3.865)(8.5) = 32.85

LCL (median moving range) = 0.00

Since Minitab does not calculate median values, the control chart is drawn by hand in Figure 8.10. The upper portion shows the new control chart for the moving ranges. The process still appears in a state of statistical control, so the median of the moving ranges may be used to construct a new individuals portion of the chart.

Figure 8.10
Individuals and Moving Range Chart (Based on Median Moving Range Value) for A-744 Batch Yields



Note that it is essential to establish stability in the moving range portion of the control chart before constructing the Individuals portion of the control chart. This is because the control limits for the Individuals portion are based on the estimate of the process variability, generated by the moving range portion. A lack of stability in the moving range portion will produce unreliable estimates of the process variation, resulting in the control chart's failure to properly separate special and common variation.

The centerline remains at the average value, 2001.63, and the control limits for the individuals portion are calculated using Equations (8.64) and (8.65):

$$\text{Centerline }(x) = 2001.63$$

$$\text{UCL }(x) = 2001.63 + (3.145)(8.5) = 2028.36$$

$$\text{LCL }(x) = 2001.63 - (3.145)(8.5) = 1974.90$$

The control chart is shown in the upper portion of Figure 8.10. There are no indications of a lack of control. Because the control limits based on the median range are narrower, they are used in this case.

8.7 Revising Control Limits for Variables Control Charts

Overly frequent revision of control limits is undesirable and inappropriate. Control limits should only be revised for one of three reasons: a change in the process; when trial control limits have been used and are to be replaced with regular control limits; and when out-of-control points have been removed from a data set due to the identification of a condition that created the out of control condition.

8.7.1 Change in Process

Processes change for many reasons. For example, such things as technical improvements, new vendors, new machines, new machine settings, new operational definitions, or new operator instructions may induce process changes. Efforts toward the never-ending reduction of variation may precipitate the change. Whatever the cause, changes in the process itself may change the variability and location and therefore necessitate a recalculation of the control limits.

8.7.2 Trial Control Limits

When control-charting is initiated for a process (for either a brand new process or an old process that is being charted for the first time), trial control limits are sometimes calculated from the first few subgroups. After about 25 or 30 subgroups become available, these trial limits should be replaced with regular control limits.

8.7.3 Removal of Out-of-Control Points

When out-of-control points used in the calculation of the control limits have been removed from a data set due to the identification and rectification of a special cause of variation, the control limits must be recalculated. As the removed data values were used to calculate the process mean, range, standard deviation, or other statistics, the removal of these data points will precipitate changes in the centerline, control limits, and zone boundaries. A new centerline and new control limits and zone boundaries must be calculated. As we have seen, this may occasionally reveal other points that are out of control and will help to further identify areas requiring some special action or changes in the process.

8.8 Collecting Data in Rational Subgroups

Proper organization of the data to be control-charted is critical if a control chart is to be helpful in improving a process. We must be certain that we're asking the right questions. In other words, the data must be organized in such a way as to permit examination of variation productively and in a manner that will reveal special sources of variation. The organization of the data defines the question the control chart is addressing to achieve process improvement. Each subgroup should be selected from a small area so that relatively homogeneous conditions exist within each subgroup; this is called rational subgrouping. Wheeler and Chambers [Wheeler, pp. 111-120] provide an excellent example of rational subgrouping.

Let us consider the case of a manufacturer of industrial paints. One-gallon cans are filled four at a time, each one by a separate filling head. The department manager is interested in learning if the weight of the product is stable and within specification and has decided to use statistical process control as an aid.

The supervisor is asked to take five successive cans from each of the four filling heads every hour. The gross weight (in kilograms) of each can is recorded for 20 measurements each hour. The supervisor continues observations for 8 consecutive hours, yielding 160 individual observations (Table 8.7).

Table 8.7

Fill Data for 160 Paint Cans

Time: 8 a.m.								**Time: 9 a.m.**						
Measurement								**Measurement**						
		1	2	3	4	5				1	2	3	4	5
H	1	6.09	6.10	6.09	6.09	6.09		**H**	1	6.13	6.13	6.14	6.13	6.11
E	2	6.09	6.09	6.10	6.09	6.09		**E**	2	6.12	6.12	6.11	6.13	6.10
A	3	6.10	6.11	6.12	6.11	6.11		**A**	3	6.11	6.13	6.13	6.14	6.14
D	4	6.16	6.16	6.17	6.17	6.17		**D**	4	6.20	6.20	6.20	6.17	6.17

Time: 10 a.m.							Time: 11 a.m.						
Measurement							Measurement						
		1	2	3	4	5			1	2	3	4	5
H	1	6.14	6.13	6.12	6.13	6.13	**H**	1	6.10	6.10	6.08	6.15	6.11
E	2	6.11	6.10	6.11	6.10	6.14	**E**	2	6.08	6.12	6.10	6.11	6.10
A	3	6.13	6.13	6.11	6.13	6.14	**A**	3	6.13	6.13	6.15	6.15	6.09
D	4	6.16	6.16	6.19	6.19	6.21	**D**	4	6.20	6.18	6.21	6.21	6.20
Time: 12 Noon							**Time: 1 p.m.**						
Measurement							**Measurement**						
		1	2	3	4	5			1	2	3	4	5
H	1	6.14	6.15	6.14	6.13	6.16	**H**	1	6.12	6.09	6.11	6.10	6.10
E	2	6.10	6.12	6.15	6.13	6.13	**E**	2	6.16	6.13	6.13	6.09	6.10
A	3	6.13	6.13	6.14	6.14	6.13	**A**	3	6.16	6.16	6.11	6.13	6.10
D	4	6.17	6.18	6.18	6.18	6.17	**D**	4	6.19	6.21	6.21	6.19	6.16
Time: 2 p.m.							**Time: 3 p.m.**						
Measurement							**Measurement**						
		1	2	3	4	5			1	2	3	4	5
H	1	6.07	6.07	6.08	6.07	6.08	**H**	1	6.11	6.12	6.13	6.13	6.13
E	2	6.07	6.08	6.07	6.07	6.09	**E**	2	6.10	6.11	6.13	6.10	6.13
A	3	6.09	6.09	6.09	6.09	6.10	**A**	3	6.13	6.16	6.14	6.13	6.13
D	4	6.15	6.15	6.16	6.16	6.14	**D**	4	6.18	6.19	6.20	6.19	6.21

How these observations are arranged may reveal variation from one of three sources: variation over time, variation between measurements, or variation between filling heads. Variation over time (hour-to-hour in this case) is represented by the differences in the groups of 20 cans; variation between measurements is represented by the differences between the five cans selected at each hour regardless of filling head; and variation between filling heads is represented by the differences between the results of the filling heads for each of the five cans selected per head, per hour.

The manager must decide on the proper arrangement of these data; their arrangement will dictate the variation that might be revealed. We will consider an arrangement of the data for each possible source of variation.

8.8.1 Arrangement 1

If the basic subgroup consists of the four head readings for a given measurement and hour, as shown in Figure 8.11, then the variation within the subgroups will be the variation from head to head. That is, our estimate of the process standard error will be based on the measurements taken across all four filling heads. The control chart will be set up to detect variation between the subgroups due to measurement-to-measurement and hour-to-hour special causes of variation. This means that the process variation is allocated as follows:

Source of Variation	Allocation
Hour-to-hour	Between subgroups
Measurement-to-measurement	Between subgroups
Head-to-head	Within subgroups

Figure 8.11
Arrangement 1

Time: 8 a.m.						
Measurement						
		1	2	3	4	5
H	1	6.09	6.10	6.09	6.09	6.09
E	2	6.09	6.09	6.10	6.09	6.09
A	3	6.10	6.11	6.12	6.11	6.11
D	4	6.16	6.16	6.17	6.17	6.17

For the eight hours, there will be a total of 40 subgroups. The average and range for each subgroup have been computed and appear in Table 8.8. The average of the averages, $\bar{\bar{x}}$, has been calculated to be 6.13. The average of the range values, $\bar{R}$, is 0.08. When the data are analyzed via this arrangement, there is some evidence of a lack of control, as shown in Figure 8.12. The evidence is the long string of points above the centerline on the $\bar{x}$ portion of the control chart (Rule 4). This, in all likelihood, results from some special source of variation that must be investigated.

Table 8.8
Arrangement 1: 40 Subgroups

Time: 8 a.m.						
Measurement						
		1	2	3	4	5
H	1	6.09	6.10	6.09	6.09	6.09
E	2	6.09	6.09	6.10	6.09	6.09
A	3	6.10	6.11	6.12	6.11	6.11
D	4	6.16	6.16	6.17	6.17	6.17
	$\bar{x}$	6.11	6.12	6.12	6.12	6.12
	R	0.07	0.07	0.08	0.08	0.08

Time: 9 a.m.						
Measurement						
		1	2	3	4	5
H	1	6.13	6.13	6.14	6.13	6.11
E	2	6.12	6.12	6.11	6.13	6.10
A	3	6.11	6.13	6.13	6.14	6.14
D	4	6.20	6.20	6.20	6.17	6.17
	$\bar{x}$	6.14	6.15	6.14	6.14	6.13
	R	0.09	0.08	0.09	0.04	0.07

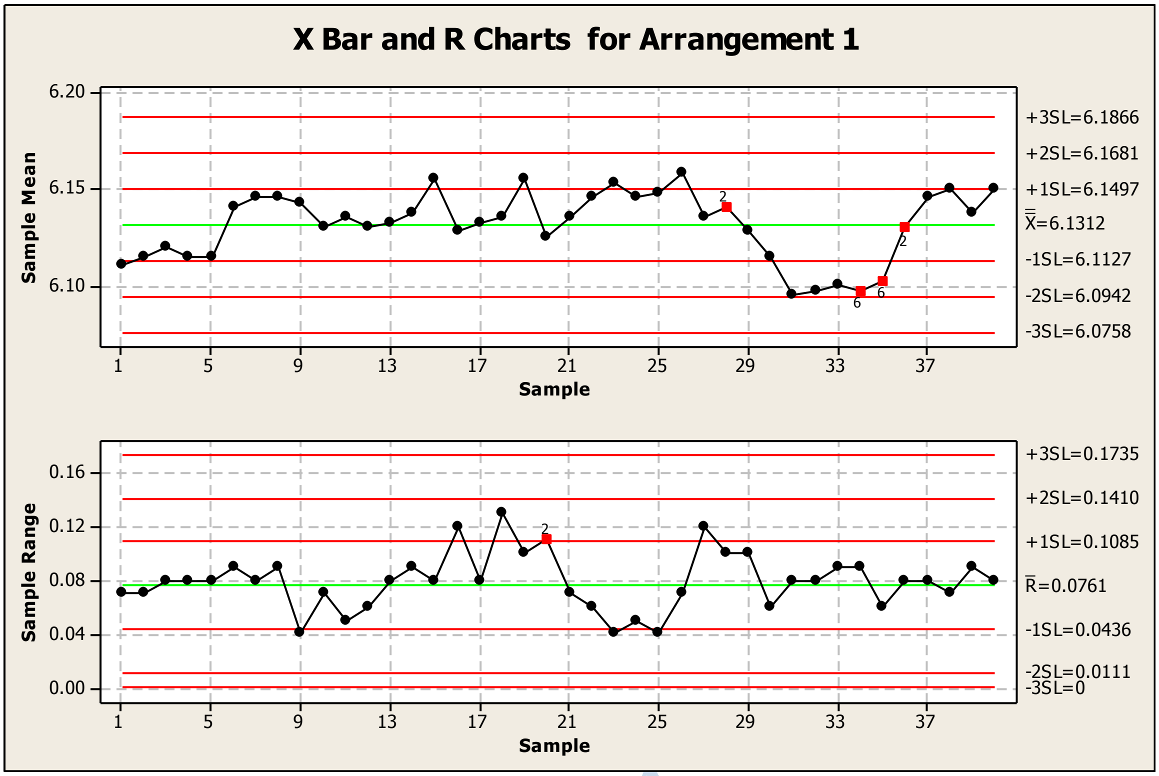
Time: 10 a.m.						
Measurement						
		1	2	3	4	5
H	1	6.14	6.13	6.12	6.13	6.13
E	2	6.11	6.10	6.11	6.10	6.14
A	3	6.13	6.13	6.11	6.13	6.14
D	4	6.16	6.16	6.19	6.19	6.21

Time: 11 a.m.						
Measurement						
		1	2	3	4	5
H	1	6.10	6.10	6.08	6.15	6.11
E	2	6.08	6.12	6.10	6.11	6.10
A	3	6.13	6.13	6.15	6.15	6.09
D	4	6.20	6.18	6.21	6.21	6.20

	$\bar{x}$	6.14	6.13	6.13	6.14	6.16
	R	0.05	0.06	0.08	0.09	0.08
Time: 12 Noon						
Measurement						
		1	2	3	4	5
H	1	6.14	6.15	6.14	6.13	6.16
E	2	6.10	6.12	6.15	6.13	6.13
A	3	6.13	6.13	6.14	6.14	6.13
D	4	6.17	6.18	6.18	6.18	6.17
	$\bar{x}$	6.14	6.15	6.15	6.15	6.15
	R	0.07	0.06	0.04	0.05	0.04
Time: 2 p.m.						
Measurement						
		1	2	3	4	5
H	1	6.07	6.07	6.08	6.07	6.08
E	2	6.07	6.08	6.07	6.07	6.09
A	3	6.09	6.09	6.09	6.09	6.10
D	4	6.15	6.15	6.16	6.16	6.14
	$\bar{x}$	6.10	6.10	6.10	6.10	6.10
	R	0.08	0.08	0.09	0.09	0.06

	$\bar{x}$	6.13	6.13	6.14	6.16	6.13
	R	0.12	0.08	0.13	0.10	0.11
Time: 1 p.m.						
Measurement						
		1	2	3	4	5
H	1	6.12	6.09	6.11	6.10	6.10
E	2	6.16	6.13	6.13	6.09	6.10
A	3	6.16	6.16	6.11	6.13	6.10
D	4	6.19	6.21	6.21	6.19	6.16
	$\bar{x}$	6.16	6.14	6.14	6.13	6.12
	R	0.07	0.12	0.10	0.10	0.06
Time: 3 p.m.						
Measurement						
		1	2	3	4	5
H	1	6.11	6.12	6.13	6.13	6.13
E	2	6.10	6.11	6.13	6.10	6.13
A	3	6.13	6.16	6.14	6.13	6.13
D	4	6.18	6.19	6.20	6.19	6.21
	$\bar{x}$	6.13	6.15	6.15	6.14	6.15
	R	0.08	0.08	0.07	0.09	0.08

Figure 8.12
Control Chart for Arrangement 1



8.8.2 Arrangement 2

If the basic subgroup consists of five measurements for a given head and hour, as shown in Table 8.9, then the variation from measurement to measurement is the basis for our estimate of the standard error. Our estimate of the process standard error is based on the observations taken across all five measurements for each one of the heads for each hour. The control chart will be set up to detect variation between subgroups due to filling-head-to-filling-head and hour-to-hour special causes of variation. This means that the process variation is allocated as follows:

Source of Variation	Allocation
Hour-to-hour	Between subgroups
Measurement-to-measurement	Within subgroups
Head-to-head	Between subgroups

Table 8.9
Arrangement 2

Time: 8 a.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.09	6.10	6.09	6.09	6.09	6.09	0.01

E	2	6.09	6.09	6.10	6.09	6.09	6.09	0.01
A	3	6.10	6.11	6.12	6.11	6.11	6.11	0.02
D	4	6.16	6.16	6.17	6.17	6.17	6.17	0.01

Time: 9 a.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.13	6.13	6.14	6.13	6.11	6.13	0.03
E	2	6.12	6.12	6.11	6.13	6.10	6.12	0.03
A	3	6.11	6.13	6.13	6.14	6.14	6.13	0.03
D	4	6.20	6.20	6.20	6.17	6.17	6.19	0.03

Time: 10 a.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.14	6.13	6.12	6.13	6.13	6.13	0.02
E	2	6.11	6.10	6.11	6.10	6.14	6.11	0.04
A	3	6.13	6.13	6.11	6.13	6.14	6.13	0.03
D	4	6.16	6.16	6.19	6.19	6.21	6.18	0.05

Time:11 a.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.10	6.10	6.08	6.15	6.11	6.11	0.07
E	2	6.08	6.12	6.10	6.11	6.10	6.10	0.04
A	3	6.13	6.13	6.15	6.15	6.09	6.13	0.06
D	4	6.20	6.18	6.21	6.21	6.20	6.20	0.03

Time:12 Noon								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.14	6.15	6.14	6.13	6.16	6.14	0.03
E	2	6.10	6.12	6.15	6.13	6.13	6.13	0.05
A	3	6.13	6.13	6.14	6.14	6.13	6.13	0.01
D	4	6.17	6.18	6.18	6.18	6.17	6.18	0.01

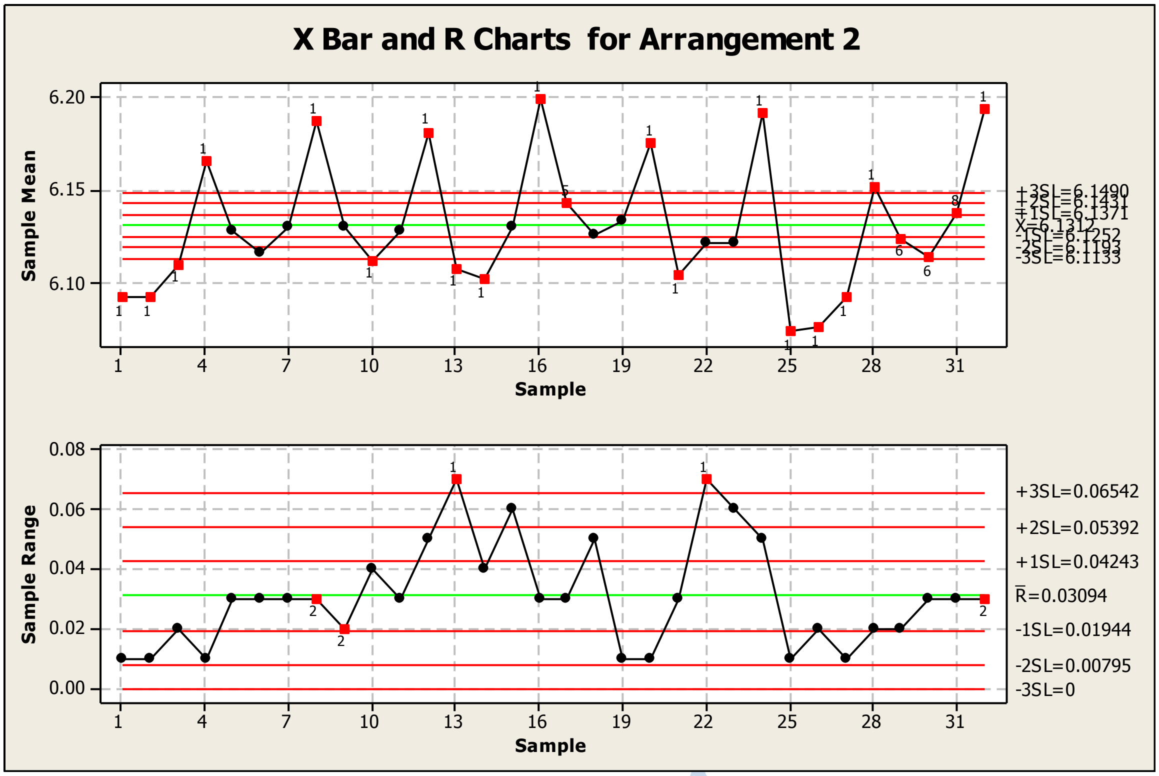
Time:1 p.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.12	6.09	6.11	6.10	6.10	6.10	0.03
E	2	6.16	6.13	6.13	6.09	6.10	6.12	0.07
A	3	6.16	6.16	6.11	6.13	6.10	6.12	0.06
D	4	6.19	6.21	6.21	6.19	6.16	6.19	0.05

Time:2 p.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.07	6.07	6.08	6.07	6.08	6.07	0.01
E	2	6.07	6.08	6.07	6.07	6.09	6.08	0.02
A	3	6.09	6.09	6.09	6.09	6.10	6.09	0.01
D	4	6.15	6.15	6.16	6.16	6.14	6.15	0.02

Time:3 p.m.								
Measurement								
		1	2	3	4	5	$\bar{x}$	R
H	1	6.11	6.12	6.13	6.13	6.13	6.12	0.02
E	2	6.10	6.11	6.13	6.10	6.13	6.11	0.03
A	3	6.13	6.16	6.14	6.13	6.13	6.14	0.03
D	4	6.18	6.19	6.20	6.19	6.21	6.19	0.03

The second arrangement has 32 subgroups each with five measurements. (Note we still have the same 160 measurements.) These five are the measurements taken each hour on each of the four filling heads. The centerline, $\bar{\bar{x}}$, remains 6.13, while the average range, $\bar{R}$, is now computed as 0.03. Figure 8.13 shows a control chart illustrating this.

Figure 8.13
Control Chart for Arrangement 2



Note that the process can now be seen as being wildly out of control, with many points beyond the control limits. Grouping the measurements by fill head reduced the within group variation so that the average range was lowered. This subsequently tightened the control limits and revealed the out-of-control points. Undoubtedly, special sources of variation are present, and the control chart indicates where we should begin our investigation; that is, careful examination reveals that many of the out-of-control points on the $\bar{x}$ portion correspond to the number 4 fill head. Its fill values are consistently above the upper control limit. Obviously fill head number 4 is putting more product on average into the containers than the other three fill heads.

The reason the overfilling of head 4 was not revealed by the first arrangement of the data and the first control chart is that the first chart was not constructed to identify differences between the fill heads; it was aimed at examining measurement-to-measurement differences and hour-to-hour differences. The second arrangement was grouped to reveal differences between the fill heads and differences from hour to hour.

8.8.3 Arrangement 3

Most revealing at this point would be a third arrangement of the data that keeps separate control charts for each fill head, as shown in Table 8.10. As each filling head has been separated with its own control chart, there is no longer any variation between the filling heads on our control chart. There are, in fact, now four distinct control charts, none of which can detect filling-head-to-filling-head variation. Computationally this third arrangement of the data is shown in Figures 8.14 (a, b, c, and d). Using this arrangement, the process variation is allocated as follows:

Source of Variation	Allocation
Hour-to-hour	Between subgroups
Measurement-to-measurement	Within subgroups

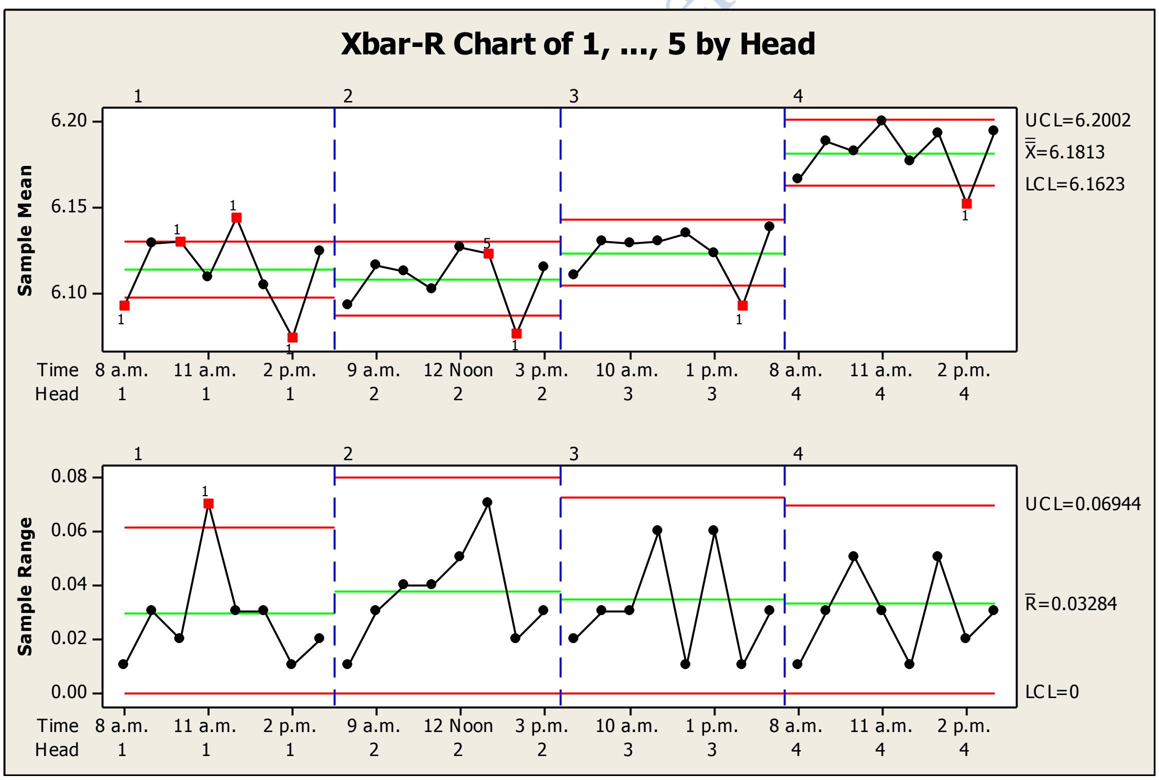
Table 8.10
Arrangement 3

Filling Head 1							
	Measurement						
Time	1	2	3	4	5	$\bar{x}$	R
8 a.m.	6.09	6.10	6.09	6.09	6.09	6.09	0.01
9 a.m.	6.13	6.13	6.14	6.13	6.11	6.13	0.03
10 a.m.	6.14	6.13	6.12	6.13	6.13	6.13	0.02
11 a.m.	6.10	6.10	6.08	6.15	6.11	6.11	0.07
12 Noon	6.14	6.15	6.14	6.13	6.16	6.14	0.03
1 p.m.	6.12	6.09	6.11	6.10	6.10	6.10	0.03
2 p.m.	6.07	6.07	6.08	6.07	6.08	6.07	0.01
3 p.m.	6.11	6.12	6.13	6.13	6.13	6.12	0.02
Filling Head 2							
	Measurement						
Time	1	2	3	4	5	$\bar{x}$	R
8 a.m.	6.09	6.09	6.10	6.09	6.09	6.09	0.01
9 a.m.	6.12	6.12	6.11	6.13	6.10	6.12	0.03
10 a.m.	6.11	6.10	6.11	6.10	6.14	6.11	0.04
11 a.m.	6.08	6.12	6.10	6.11	6.10	6.10	0.04
12 Noon	6.10	6.12	6.15	6.13	6.13	6.13	0.05
1 p.m.	6.16	6.13	6.13	6.09	6.10	6.12	0.07
2 p.m.	6.07	6.08	6.07	6.07	6.09	6.08	0.02
3 p.m.	6.10	6.11	6.13	6.10	6.13	6.11	0.03
Filling Head 3							
	Measurement						
Time	1	2	3	4	5	$\bar{x}$	R
8 a.m.	6.10	6.11	6.12	6.11	6.11	6.11	0.02
9 a.m.	6.11	6.13	6.13	6.14	6.14	6.13	0.03
10 a.m.	6.13	6.13	6.11	6.13	6.14	6.13	0.03
11 a.m.	6.13	6.13	6.15	6.15	6.09	6.13	0.06
12 Noon	6.13	6.13	6.14	6.14	6.13	6.13	0.01
1 p.m.	6.16	6.11	6.11	6.13	6.10	6.12	0.06
2 p.m.	6.09	6.09	6.09	6.09	6.10	6.09	0.01
3 p.m.	6.13	6.16	6.14	6.13	6.13	6.14	0.03
Filling Head 4							
	Measurement						

Time	1	2	3	4	5	$\bar{x}$	R
8 a.m.	6.16	6.16	6.17	6.17	6.17	6.17	0.01
9 a.m.	6.20	6.20	6.20	6.17	6.17	6.19	0.03
10 a.m.	6.16	6.16	6.19	6.19	6.21	6.18	0.05
11 a.m.	6.20	6.18	6.21	6.21	6.20	6.20	0.03
12 Noon	6.17	6.18	6.18	6.18	6.17	6.18	0.01
1 p.m.	6.19	6.21	6.21	6.19	6.16	6.19	0.05
2 p.m.	6.15	6.15	6.16	6.16	6.14	6.15	0.02
3 p.m.	6.18	6.19	6.20	6.19	6.21	6.19	0.03

Note that head-to-head variation is no longer within the control chart and is only visible by comparing the different control charts. Arranging the data in this way permits the construction of individual sets of $\bar{x}$ and R charts for each filling head. When the four head control charts are drawn on the same scale, as in Figure 8.14, the charts reveal things that may have been obscured earlier: head 4 is significantly different from heads 1, 2, and 3.This information allows management to take appropriate action on the process, that is, fix head 4.

Figure 8.14
Control Charts for All 4 Heads



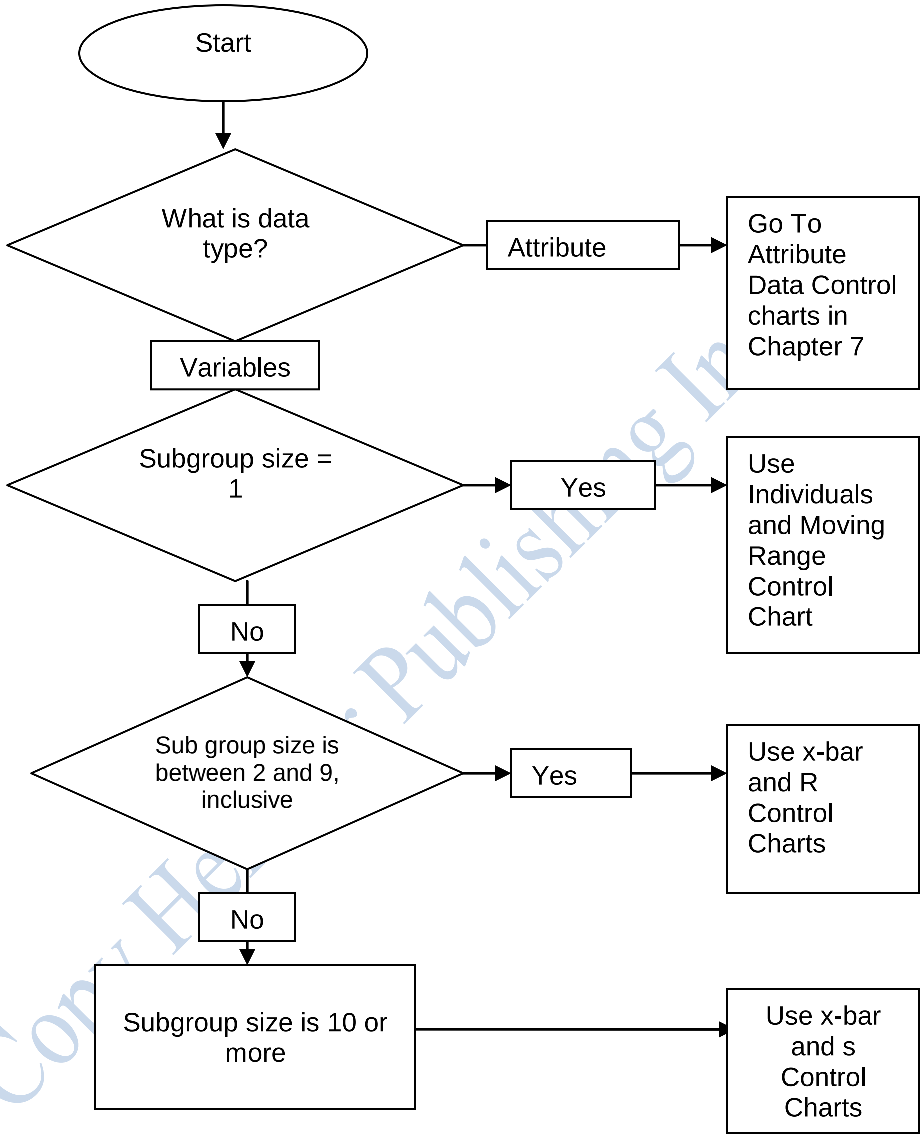
Thus, proper subgrouping of the data to be control-charted is critical to process improvement efforts. Knowledge of the process under study is often the best guide to rational subgrouping of data.

8.9 Summary

Variables (measurement) data consist of numerical measurements such as cycle time, waiting time, revenue, cost, weight, length, temperature, and electrical resistance. Variables data contain more information than attribute data because they measure a characteristic over a continuous scale (as opposed to conforming or non-conforming), or they measure the difference between a characteristic and its desired (nominal) value, even within specification limits. Measurement data reduce variation in a process, even if all units of output are within specification limits. This is the Taguchi Loss Function view of quality; it results in the never-ending reduction of variation in a process. Measurement data use all the information contained in the data; this alone makes variables charts preferable to attribute charts when a choice is possible.

There are three principal types of variables control charts: the $\bar{x}$ and R chart, the $\bar{x}$ and s chart, and the individuals and moving range chart. All are used in the never-ending spiral of process improvement.

The following flowchart presents a pictorial view of the different types of variables control charts in relation to subgroup size.



The $\bar{x}$ and R chart uses the subgroup range, R, to chart the process variability, and the subgroup average, $\bar{x}$, to chart the process location when subgroups consist of between two and nine measurements. Stable processes yield subgroups that will behave predictably, enabling us to construct an $\bar{x}$ and R chart.

$\bar{x}$ and s charts use the standard deviation to chart the process variability, and the subgroup average, $\bar{x}$, to chart the process location when subgroups consist of 10 or more observations.

Individuals and Moving Range charts are used when only a single variable value is measured per subgroup. Single measurements of variables are considered a subgroup of size one.

In general, subgroups should be large enough to detect points or patterns indicating a lack of control when a lack of control exists. The frequency with which subgroups are selected depends upon the speed with which a process can change. Processes that can change frequently require frequent selection of subgroups; while processes that change slowly require less frequent selection of subgroups.

Frequent regular revision of control limits is undesirable and inappropriate. Control limits should be revised only for one of three reasons: a change in the process; when trial control limits have been used and are to be replaced with regular control limits; and when points out of control have been eliminated from a data set.

Proper organization of the data to be control-charted is critical if a control chart is to be helpful in process improvement. In other words, the data must be organized in such a way as to permit examination of variation productively and in a manner that will reveal special sources of variation. Rational subgrouping, or the organization of the data, defines the question the control chart is addressing to achieve process improvement.

EXERCISES

8.1 The housekeeping staff in a large resort hotel cleans and prepares all of the guestrooms daily. In an effort to improve service through reducing variation in the time required to clean and prepare a room, a series of measurements is taken of the times to service rooms in one section of the hotel. The cleaning times for five rooms for 25 consecutive days are shown below, in minutes:

HOTEL

Room	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Day 8**	**Day 9**	**Day 10**
1	15.6	15.0	16.4	14.2	16.4	14.9	17.9	14.0	17.6	14.6
2	14.3	14.8	15.1	14.8	16.3	17.2	17.9	17.7	16.5	14.0
3	17.7	16.8	15.7	17.3	17.6	17.2	14.7	16.9	15.3	14.7
4	14.3	16.9	17.3	15.0	17.9	15.3	17.0	14.0	14.5	16.9
5	15.0	17.4	16.6	16.4	14.9	14.1	14.5	14.9	15.1	14.2
Room	**Day 11**	**Day 12**	**Day 13**	**Day 14**	**Day 15**	**Day 16**	**Day 17**	**Day 18**	**Day 19**	**Day 20**

1	14.6	15.3	17.4	15.3	14.8	16.1	14.2	14.6	15.9	16.2
2	15.5	15.3	14.9	16.9	15.1	14.6	14.7	17.2	16.5	14.8
3	15.9	15.9	17.7	17.7	16.6	17.5	15.3	16.0	16.1	14.8
4	14.8	15.0	16.6	17.2	16.3	16.9	15.7	16.7	15.0	15.0
5	14.2	17.8	14.7	17.5	14.5	17.7	14.3	16.3	17.8	15.3
Room	**Day 21**	**Day 22**	**Day 23**	**Day 24**	**Day 25**					
1	16.3	15.0	16.4	16.6	17.0					
2	15.3	17.6	15.9	15.1	17.5					
3	14.0	14.5	16.7	14.1	17.4					
4	17.4	17.5	15.7	17.4	16.2					
5	14.5	17.8	16.9	17.8	17.9					

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.2 A parcel-sorting facility's management wants to know how much time is needed to sort units in what is termed the primary sort. The variable to be measured is the number of units sorted in a one-minute interval by a given team. Each hour, the first five one-minute intervals are selected. The results are shown below:

SORTED

Subgroup Number	Observation				
	1	2	3	4	5
1	474	386	528	333	465
2	688	367	691	602	569
3	427	500	279	479	721
4	846	506	420	509	409
5	384	365	884	521	562
6	774	889	326	581	270
7	728	902	809	468	318
8	682	614	315	68	594
9	401	418	429	72	781
10	613	394	421	684	675
11	874	546	804	709	469
12	409	591	665	685	540
13	748	818	694	481	290
14	790	796	705	503	710
15	227	409	621	495	891

16	588	789	344	785	724
17	725	802	336	645	782
18	671	691	735	351	853
19	778	795	462	301	549
20	366	691	644	547	705

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.3 The drive-up window at a local bank is searching for ways to improve service. One teller keeps a control chart for the service time in minutes for the first four customers driving up to her window each hour for a three-day period. The results of her data collection are shown below:

TELLERTIME

Cust.	**Time**							
	9 a.m.	**10 a.m.**	**11 a.m.**	**12 noon**	**1 p.m.**	**2 p.m.**	**3 p.m.**	**4 p.m.**
1	1.4	3.8	3.6	4.3	4.0	1.3	0.9	4.7
2	2.3	5.2	2.5	1.2	5.2	1.1	4.4	5.1
3	1.9	1.9	0.8	3.0	2.7	4.9	5.1	0.9
4	5.1	4.8	2.9	1.5	0.3	2.3	4.6	4.7

Cust.	**Time**							
	9 a.m.	**10 a.m.**	**11 a.m.**	**12 noon**	**1 p.m.**	**2 p.m.**	**3 p.m.**	**4 p.m.**
1	2.8	0.5	4.5	0.6	4.8	2.7	4.2	0.9
2	3.0	2.7	1.9	1.2	2.8	2.0	1.1	4.4
3	4.1	4.7	4.2	2.7	1.1	2.6	4.4	0.6
4	4.8	3.6	0.4	2.5	0.4	2.6	3.1	0.4

Cust.	**Time**							
	9 a.m.	**10 a.m.**	**11 a.m.**	**12 noon**	**1 p.m.**	**2 p.m.**	**3 p.m.**	**4 p.m.**
1	0.3	3.5	5.2	2.9	3.3	4.0	2.8	0.6
2	2.4	3.4	0.3	1.9	3.7	3.3	0.7	2.1
3	5.0	4.6	2.4	0.8	3.8	5.0	1.6	3.3
4	0.9	3.3	3.9	0.3	2.1	2.8	4.6	2.7

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?

c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.4 A paper products manufacturer coats one particular paper product with wax. In an effort to control and stabilize the coating process, the employee running the coating machine takes six measurements of coating thickness every 15 minutes during a one-day study period. The results of the data collection are shown below:

Coating Thickness

	Time								
	8 am	**8:15 am**	**8:30 am**	**8:45 am**	**9 am**	**9:15 am**	**9:30 am**	**9:45 am**	**10 am**
$\bar{x}$	2.90	3.30	2.87	0.93	1.43	1.93	2.90	3.03	3.70
R	0.35	0.50	0.32	0.44	0.91	0.42	0.29	0.78	1.01
	Time								
	10:15 am	**10:30 am**	**10:45 am**	**11 am**	**11:15 am**	**11;30 am**	**11:45 am**	**12 noon**	**12:15 pm**
$\bar{x}$	3.70	3.00	3.20	2.80	2.90	3.53	3.90	3.73	3.23
R	1.09	0.95	0.56	0.42	0.55	0.16	1.04	0.90	0.86
	Time								
	12:30 pm	**12:45 pm**	**1 pm**	**1:15 pm**	**1:30 pm**	**1:45 pm**	**2 pm**	**2:15 pm**	**2:30 pm**
$\bar{x}$	2.73	3.00	3.33	2.50	2.13	1.97	2.70	3.10	2.63
R	0.19	0.84	0.20	0.71	0.90	0.92	1.00	0.56	0.88
	Time								
	2:45 pm	**3 pm**	**3:15 pm**	**3:30 pm**	**3:45 pm**	**4 pm**			
$\bar{x}$	2.63	3.40	3.20	2.20	2.03	3.23			
R	0.89	1.06	0.66	0.93	0.59	0.91			

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.5 Groups of customers arriving at a restaurant must wait to be seated by a hostess. Waiting time is short if a table is available, but long if the restaurant is crowded. The hostess records waiting times for the first eight people arriving from 7:00 pm each Friday, Saturday, and Sunday evening for a series of 10 consecutive weeks. She then computes the average and range for each subgroup. The results of the data collection and calculations are shown below:

Waiting Times (minutes)

	Date									
	9/2	**9/3**	**9/4**	**9/9**	**9/10**	**9/11**	**9/16**	**9/17**	**9/18**	**9/23**
$\bar{x}$	15.5	23.0	18.0	11.5	12.0	12.3	15.5	13.5	21.5	12.2
R	5.1	6.2	2.6	2.4	4.8	2.6	2.8	4.0	4.8	1.8
	Date									
	9/24	**9/25**	**9/30**	**10/1**	**10/2**	**10/7**	**10/8**	**10/9**	**10/14**	**10/15**
$\bar{x}$	21.5	28.0	16.0	17.0	31.0	21.1	15.0	17.5	17.5	18.0
R	4.0	3.6	3.0	4.4	7.2	2.1	3.6	1.8	1.8	3.6
	Date									
	10/16	**10/21**	**10/22**	**10/23**	**10/28**	**10/29**	**10/30**	**11/4**	**11/5**	**11/6**
$\bar{x}$	17.5	20.2	26.0	17.0	19.0	20.0	16.0	18.0	18.5	19.5
R	3.6	4.8	5.6	3.4	3.4	3.6	3.2	3.8	3.4	3.6

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.6 The production department of a newspaper has embarked upon a quality improvement effort. After several brainstorming sessions, the team has chosen as its first project an issue that relates to the blackness of the print of the newspaper: each day a determination needs to be made concerning how “black” the newspaper print is. This is measured on a densitometer that records the results on a standard scale. Each day, five spots on the first newspaper printed are chosen and the blackness of each spot is measured. The results for 20 consecutive weekdays are presented in the table below.

BLACK

	Spot				
Day	**1**	**2**	**3**	**4**	**5**
1	0.96	1.01	1.12	1.07	0.97
2	1.06	1.00	1.02	1.16	0.96
3	1.00	0.90	0.98	1.18	0.96
4	0.92	0.89	1.01	1.16	0.90
5	1.02	1.16	1.03	0.89	1.00
6	0.88	0.92	1.03	1.16	0.91
7	1.05	1.13	1.01	0.93	1.03
8	0.95	0.86	1.14	0.90	0.95
9	0.99	0.89	1.00	1.15	0.92
10	0.89	1.18	1.03	0.96	1.04
11	0.97	1.13	0.95	0.86	1.06

12	1.00	0.87	1.02	0.98	1.13
13	0.96	0.79	1.17	0.97	0.95
14	1.03	0.89	1.03	1.12	1.03
15	0.96	1.12	0.95	0.88	0.99
16	1.01	0.87	0.99	1.04	1.16
17	0.98	0.85	0.99	1.04	1.16
18	1.03	0.82	1.21	0.98	1.08
19	1.02	0.84	1.15	0.94	1.08
20	0.90	1.02	1.10	1.04	1.08

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.7 The manager of a branch of a local bank wants to study waiting times of customers for teller service during the peak 12 noon to 1 P.M. lunch hour. A subgroup of four customers is selected (one at each 15-minute interval during the hour), and the time in minutes is measured from the point each customer enters the line to when His or her service begins. The results over a 4-week period are as follows:

BANKTIME

Day	Time in Minutes			
1	7.2	8.4	7.9	4.9
2	5.6	8.7	3.3	4.2
3	5.5	7.3	3.2	6.0
4	4.4	8.0	5.4	7.4
5	9.7	4.6	4.8	5.8
6	8.3	8.9	9.1	6.2
7	4.7	6.6	5.3	5.8
8	8.8	5.5	8.4	6.9
9	5.7	4.7	4.1	4.6
10	1.7	4.0	3.0	5.2
11	2.6	3.9	5.2	4.8
12	4.6	2.7	6.3	3.4
13	4.9	6.2	7.8	8.7
14	7.1	6.3	8.2	5.5
15	7.1	5.8	6.9	7.0
16	6.7	6.9	7.0	9.4
17	5.5	6.3	3.2	4.9
18	4.9	5.1	3.2	7.6
19	7.2	8.0	4.1	5.9
20	6.1	3.4	7.2	5.9

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.8 The Director of Radiology at a large metropolitan hospital is concerned about the scheduling of the radiology facilities. 250 patients per day, on average, are transported from wards to the Radiology Department for treatment or diagnostic procedures. If patients do not reach their radiology unit at their scheduled time, backups will occur and other patients will experience delays. The time it takes to transport patients from wards to the radiology unit was operationally defined as the time between when the transporter was assigned to the patient and the time the patient arrived at the radiology unit. A sample of n = 4 patients was selected each day for 20 days and the time to transport each patient (in minutes) was measured, with the following results:

TRANSPORT

	Patient			
Day	**1**	**2**	**3**	**4**
1	16.3	17.4	18.7	16.9
2	29.4	17.3	22.7	10.9
3	12.2	12.7	14.1	10.3
4	22.4	19.7	24.9	23.4
5	13.5	11.6	14.8	13.5
6	15.2	23.6	19.4	20.0
7	23.1	13.6	21.1	13.7
8	15.7	10.9	16.4	21.8
9	10.2	14.9	12.6	11.9
10	14.7	18.7	22.0	19.1
11	15.6	19.1	22.9	19.4
12	19.8	12.2	26.7	19.0
13	24.3	18.7	30.3	22.9
14	16.5	14.3	19.5	15.5
15	23.4	27.6	30.7	24.0
16	9.7	14.6	10.4	10.8
17	27.8	18.4	23.7	22.8
18	17.4	25.8	18.4	9.0
19	20.5	17.8	23.2	18.0
20	14.2	14.6	11.1	17.7

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.9 The Telecommunications Department for a county General Services Agency, is responsible for the repair of equipment used in radio communications by police, fire, and emergency medical services in the county. The timely repair of the radios is critically important for the efficient operation of these public service units. The following table shows the repair times in minutes for a daily sample of five radios taken over a 30-day period:

RADIO

	Radio				
Day	**1**	**2**	**3**	**4**	**5**
1	114	499	106	342	55
2	219	319	162	44	87
3	64	302	38	83	93
4	258	110	98	78	154
5	127	140	298	518	275
6	151	176	188	268	77
7	24	183	202	81	104
8	41	249	342	338	69
9	93	189	209	444	151
10	111	207	143	318	129
11	205	281	250	468	79
12	121	261	183	606	287
13	225	83	198	223	180
14	235	439	102	330	190
15	91	32	190	70	150
16	181	191	182	444	124
17	52	190	310	245	156
18	90	538	277	308	171
19	78	587	147	172	299
20	45	265	126	137	151
21	410	227	179	298	342
22	68	375	195	67	72
23	140	266	157	92	140
24	145	170	231	60	191
25	129	74	148	119	139
26	143	384	263	147	131
27	86	229	474	181	40
28	164	313	295	297	280
29	257	310	217	152	351
30	106	134	175	153	69

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?

c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.10 The workers in a packaging operation shovel 30 kilograms of a granular product from a large pile into sacks that are then sealed and placed on pallets for shipping. The scale used by the workers is accurate, and an effort has been made to educate the workers about the need to measure each weight carefully. A sequence of 25 subgroups, each consisting of the weights of five sacks, has been recorded.

PACKAGING

Weight (kilograms)

Subgroup Number	**Observation**				
	1	**2**	**3**	**4**	**5**
1	35.4	35.6	34.8	34.7	34.8
2	36.0	35.6	34.9	34.8	35.9
3	35.2	35.0	35.0	35.4	35.1
4	34.8	35.8	35.2	35.0	34.9
5	34.2	35.0	36.1	34.9	35.1
6	36.0	35.0	35.2	34.8	34.9
7	36.1	34.9	34.5	35.0	35.1
8	35.1	35.0	35.6	34.9	36.2
9	35.0	35.6	36.1	34.8	35.6
10	35.4	35.8	36.0	34.2	36.0
11	35.2	35.3	35.2	35.9	34.8
12	35.9	36.0	35.1	35.1	35.6
13	35.2	35.6	35.0	34.9	35.0
14	35.2	35.6	35.8	35.0	35.1
15	34.9	34.8	35.0	35.2	34.9
16	35.2	35.3	35.2	35.6	35.1
17	35.6	35.8	35.2	35.4	34.9
18	35.2	35.6	35.4	35.6	35.2
19	34.7	34.9	35.6	35.2	35.0
20	35.0	35.1	35.6	35.0	35.1
21	35.6	35.0	35.8	35.2	34.6
22	34.9	35.1	35.6	35.0	35.2
23	35.9	36.0	35.2	36.0	35.2
24	35.2	35.1	35.4	34.9	35.1
25	35.2	35.2	35.1	35.6	34.9

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?

c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.11 A hospital administrator is studying how long an emergency room patient waits to see a physician during the midnight to 8:00 am shift. The study is limited to those patients who actually do see a physician, and the length of waiting time has been carefully operationally defined. Each day, the first 15 records are studied, with the results shown below:

ERWAIT

Waiting Time (minutes)

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
2	26	19	24	40	31	39	16
32	2	18	33	22	13	41	24
8	40	15	46	23	15	8	27
30	17	18	20	40	4	37	17
38	12	18	32	34	35	17	13
24	14	18	5	7	40	48	30
31	9	44	48	23	41	36	44
46	45	3	20	37	39	32	23
49	13	28	39	31	31	40	50
32	43	47	2	48	17	30	41
32	42	44	16	23	18	38	18
10	33	3	12	45	32	22	14
43	7	13	36	15	8	1	30
27	4	37	47	43	30	33	9
41	34	5	40	5	28	13	44

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.12 The food services director of an airline wants to measure the weight of food left over on passenger food trays to measure the difference between product performance and customer expectations. Each day, 12 of the trays returned in the main cabin on a particular flight are set aside, placed into a special container, and sent to a lab for weighing. The average and standard deviation of each of these subgroups over a period of 25 days is shown below.

Weight of Leftovers (grams)

	Day									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$\bar{x}$	44.5	61.5	66.6	64.9	22.9	12.7	44.7	10.4	82.6	55.2
s	23.5	11.2	28.6	31.9	13.3	12.5	32.8	35.1	39.1	10.9
	Day									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\bar{x}$	92.9	25.5	92.9	27.5	84.8	82.2	24.7	25.8	79.7	19.5
s	35.5	38.3	27.5	21.3	22.5	35.2	20.7	19.4	14.7	23.5
	Day									
	21	**22**	**23**	**24**	**25**					
$\bar{x}$	10.9	47.5	72.2	28.6	90.0					
s	36.7	28.6	25.9	22.7	23.4					

a. Manually construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.13 A hospital is studying the length of time patients spend in their routine admitting procedure. Samples of 12 are selected each day for a 20-day period. Admitting time (which has been carefully operationally defined) is measured in seconds, with the following subgroup results:

ADMITTING

Admitting Time (seconds)

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
362	611	320	621	680	759	372	370	530	494
468	873	944	927	794	665	835	294	881	914
553	768	593	948	650	730	884	480	943	870
390	807	857	817	780	930	930	558	383	272
460	476	710	641	442	369	667	502	316	662
910	816	724	764	372	635	747	595	611	348
707	567	545	986	627	313	390	847	778	447
829	833	526	430	882	843	644	544	531	306
955	521	348	743	756	264	339	853	896	751
705	959	456	451	548	663	664	876	772	415
884	315	576	645	767	991	245	744	719	717
904	414	855	996	745	431	893	816	670	387

Day 11	Day 12	Day 13	Day 14	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20
659	274	797	678	997	242	594	368	806	497
919	754	253	679	205	474	817	850	575	785
603	428	829	351	893	966	381	510	348	806
897	811	857	663	734	823	42	688	298	263
319	916	898	638	474	515	429	201	487	435
499	332	387	928	631	617	786	795	697	337
799	765	918	258	746	894	901	977	249	659
482	961	900	338	642	519	278	715	668	537
615	437	691	446	484	636	472	253	533	786
497	692	600	936	525	547	885	310	985	607
430	80	450	584	685	993	991	412	284	466
765	566	775	535	358	858	557	813	707	564

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.14 Specifications for toothpaste call for the amount of active ingredient in each sample to be 7.20mg +/- 0.08mg. Ten samples are drawn each day and the amount of active ingredient in each sample is determined. The table below shows the mean amount and the standard deviation for 30 samples:

Day	Mean	Standard Deviation
1	7.14	.163
2	7.14	.117
3	7.21	.057
4	7.18	.162
5	7.14	.117
6	7.22	.092
7	7.13	.074
8	7.20	.067
9	7.26	.052
10	7.20	.082
11	7.25	.053
12	7.17	.134
13	7.25	.071
14	7.24	.084
15	7.19	.099
16	7.16	.126
17	7.21	.089
18	7.24	.097

19	7.17	.142
20	7.16	.143
21	7.25	.071
22	7.15	.127
23	7.19	.159
24	7.13	.082
25	7.25	.097
26	7.18	.103
27	7.24	.084
28	7.19	.088
29	7.24	.097
30	7.26	.107

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.15 A manufacturer of a special chemical fertilizer is concerned with the pH of finished batches of product. A series of careful measurements reveals:

PH

Batch	pH	Batch	pH	Batch	pH	Batch	pH
1	6.5	26	6.1	51	6.6	76	6.4
2	6.2	27	6.8	52	6.1	77	6.7
3	6.7	28	6.6	53	6.3	78	6.4
4	6.7	29	6.4	54	6.8	79	6.2
5	.6.2	30	6.0	55	6.3	80	6.1
6	6.1	31	6.5	56	6.7	81	6.4
7	6.8	32	6.0	57	6.1	82	6.2
8	6.4	33	6.5	58	6.6	83	6.7
9	6.0	34	6.8	59	6.6	84	6.5
10	6.6	35	6.2	60	6.6	85	6.8
11	6.7	36	6.0	61	6.7	86	6.2
12	6.9	37	6.6	62	6.2	87	6.7
13	6.2	38	6.3	63	6.5	88	6.5
14	6.9	39	6.2	64	6.0	89	6.1
15	6.1	40	6.7	65	6.8	90	6.5
16	6.5	41	6.2	66	6.2	91	6.7
17	6.3	42	6.4	67	6.0	92	6.0
18	.2	43	6.3	68	6.1	93	6.0
19	6.9	44	6.7	69	6.8	94	6.6
20	6.6	45	6.1	70	6.7	95	6.2

21	6.7	46	6.3	71	6.8	96	6.4
22	6.6	47	6.0	72	6.3	97	6.1
23	6.3	48	6.6	73	6.6	98	6.5
24	6.6	49	6.8	74	6.3	99	6.1
25	6.2	50	6.5	75	6.6	100	6.9

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Do the control limits appear inflated? Explain.
d. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.16 A small independent public water utility in the San Francisco area monitored daily usage of water (1 unit = 748,000 gallons) for a period of ten weeks (Monday to Friday only) during September, October, and November in a recent year. The following table shows the daily water usage for 50 weekdays. Records show that on day 5 a water main broke and a major leak occurred. No other anomalies were recorded.

WATER

Day	Water Usage
1	2.50
2	2.67
3	2.73
4	2.64
5	4.45
6	2.74
7	3.31
8	2.49
9	2.76
10	2.63
11	2.47
12	2.59
13	3.20
14	3.02
15	2.31
16	2.30
17	2.50
18	3.31
19	2.48
20	2.22
21	2.61
22	2.64

23	2.92
24	2.10
25	3.18
26	2.39
27	3.51
28	2.23
29	2.55
30	2.64
31	2.58
32	2.98
33	2.43
34	2.39
35	3.22
36	2.33
37	2.27
38	2.30
39	2.29
40	2.20
41	3.31
42	3.45
43	3.23
44	2.29
45	2.33
46	2.34
47	3.31
48	2.31
49	2.64
50	2.24

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.17 One of the important quality characteristics of paste ink used in lithographic printing presses is viscosity. For ink to flow properly through the press and be applied to the paper, it must be within a limited range of viscosity. If the viscosity is too high, the ink will not flow through the press fast enough, while too low a viscosity value will result in too much ink flowing through the press. Either of these conditions will adversely affect the quality of the printed material. A viscosity measure is taken at the end of production of each batch of ink. Viscosity measures for 50 consecutive batches are shown below:

PASTEINK

Sample	Viscosity	Sample	Viscosity

1	305	26	291
2	274	27	301
3	290	28	290
4	314	29	290
5	291	30	308
6	315	31	306
7	301	32	292
8	298	33	279
9	306	34	276
10	305	35	285
11	270	36	296
12	296	37	275
13	307	38	299
14	284	39	301
15	280	40	294
16	264	41	312
17	299	42	289
18	270	43	278
19	275	44	288
20	276	45	299
21	294	46	270
22	313	47	308
23	304	48	298
24	310	49	294
25	271	50	306

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

8.18 The following data represent the amount of soft drink filled in a subgroup of 50 consecutive two-liter bottles. The nominal fill amount is 2.0 liters ± 0.11 liters. The results, listed horizontally in the order of being filled, were:

DRINK

2.109	2.086	2.066	2.075	2.065
2.057	2.052	2.044	2.036	2.038
2.031	2.029	2.025	2.029	2.023
2.020	2.015	2.014	2.013	2.014
2.012	2.012	2.012	2.010	2.005
2.003	1.999	1.996	1.997	1.992
1.994	1.986	1.984	1.981	1.973

1.975	1.971	1.969	1.966	1.967
1.963	1.957	1.951	1.951	1.947
1.941	1.941	1.938	1.908	1.894

a. Construct a control chart for the data above. Use zone boundaries.
b. Is the process in control? If not, when does the out-of-control behavior occur?
c. Drop the out-of-control points (assuming you have discovered any and changed the process to resolve the special causes) and recompute the control chart. Is the process now in control? If not, when does the out-of-control behavior occur?

REFERENCES AND ADDITIONAL READINGS

[1] W. A. Shewhart, Economic Control of Quality of Manufactured Product (New York: D. Van Nostrand, 1931).

[2] Donald J. Wheeler and David S. Chambers, Understanding Statistical Process Control (Knoxville, Tenn.: Statistical Process Controls, 1986).

Appendix A8.1
Using Minitab for Variables Charts

Note for Minitab (version 16): The authors recommend that before creating either x-bar and R or x-bar and s charts, the following steps should be taken. Go to the Tools tab – select Options – open Control charts and Quality Tools – select estimating the standard deviation – for R and xbar –R charts make sure that Rbar is selected and for s and xbar-s charts make sure that Sbar is selected. This is a change to the default settings in Minitab 16.

Using Minitab for the $\overline{X}$ and R Charts

$\overline{X}$ and R charts can be obtained from Minitab by selecting **Stat | Control Charts | Variable Charts for Subgroups | Xbar-R** from the menu bar. The format for entering the variable name is different, depending on whether the data are stacked down a single column or unstacked across a set of columns with the data for each time period in a single row. If the data for the variable of interest are stacked down a single column, choose Single Column in the Data are arranged as drop-down list box and enter the variable name in the edit box below. If the subgroups are unstacked with each row representing the data for a single time period, choose Subgroups as rows in the Data are arranged as drop-down list box and enter the variable names for the data in the edit box below.

To illustrate how to obtain $\overline{X}$ and R charts, refer to the data of Table 8.1 concerning the weight of vials. Open the **VIALS.MTW** worksheet.

Select **Stat | Control Charts | Variable Charts for Subgroups | Xbar-R**. In the Xbar-R Chart – Select the pull down option of "Observations for a Subgroup are in one row of columns:" and click in the box just below to display the list of columns as shown in Figure A8.1. Select or enter the desired column numbers (here **C1 to C6).** Click the Xbar-R Options button

1. In the Xbar-R Chart Options dialog box, pull down to the "perform all tests for special causes" option as shown in Figure A8.2. Click the **OK** button to return to the Xbar-R Data Source Chart dialog box. These values will stay intact until Minitab is restarted.
2. If there are points that should be omitted when estimating the center line and control limits, click the **Data Options** tab in the Xbar-R Chart Options dialog box, as shown in Figure A8.3. Enter the

subgroups to be omitted in the edit box shown. Click the **OK** button to return to the Xbar-R Chart dialog box.

Figure A8.1
Minitab Xbar-R Data Source Chart Dialog Box

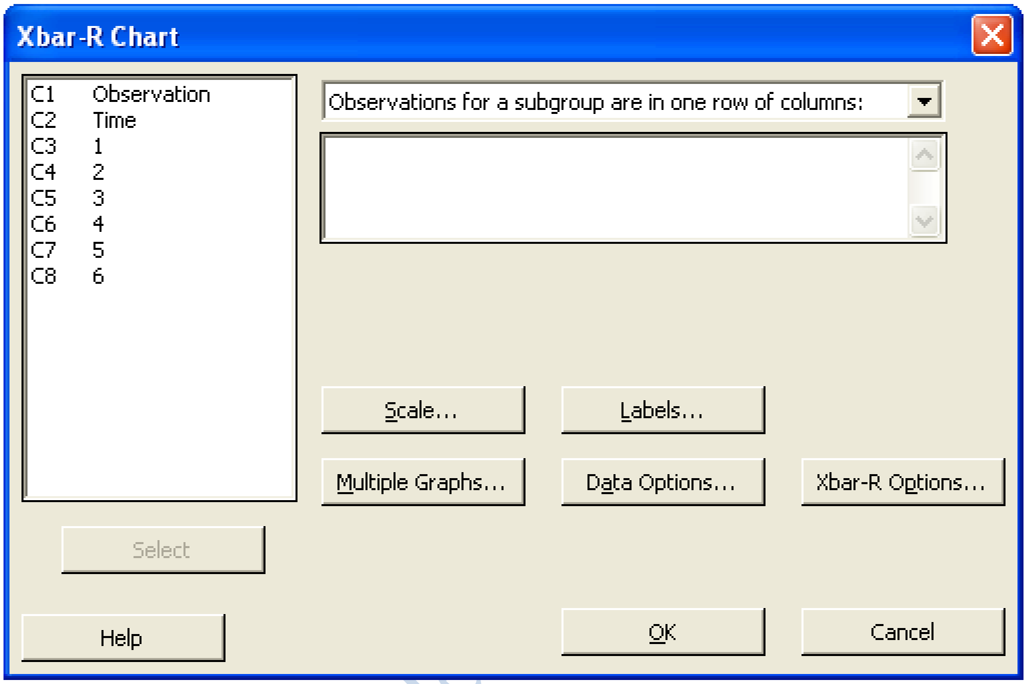


Figure A87.2
Minitab Xbar-R Chart Options Dialog Box, Tests Tab

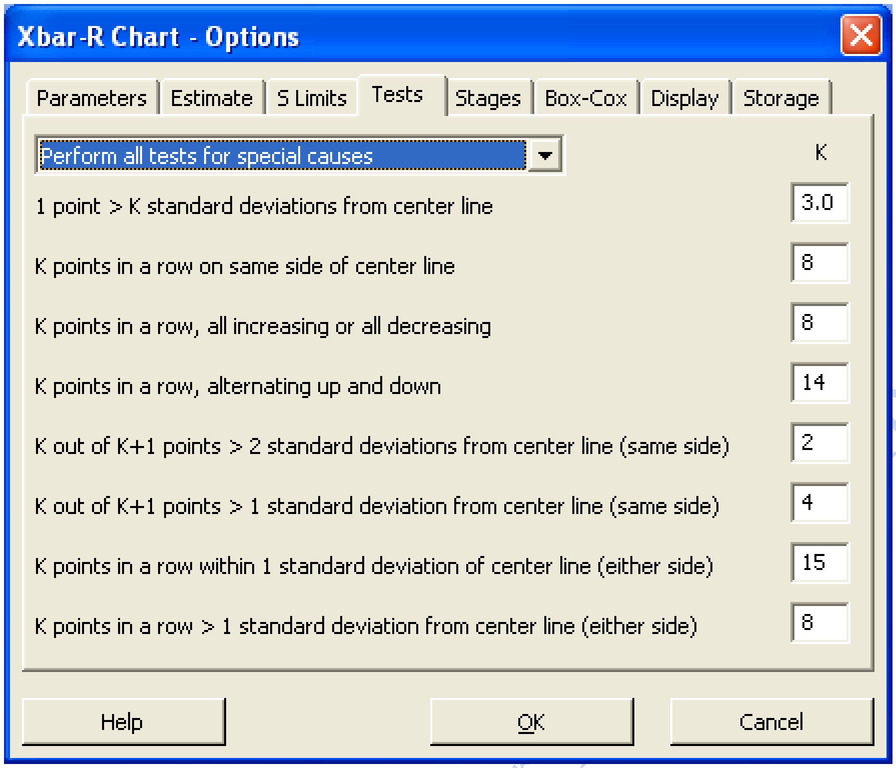
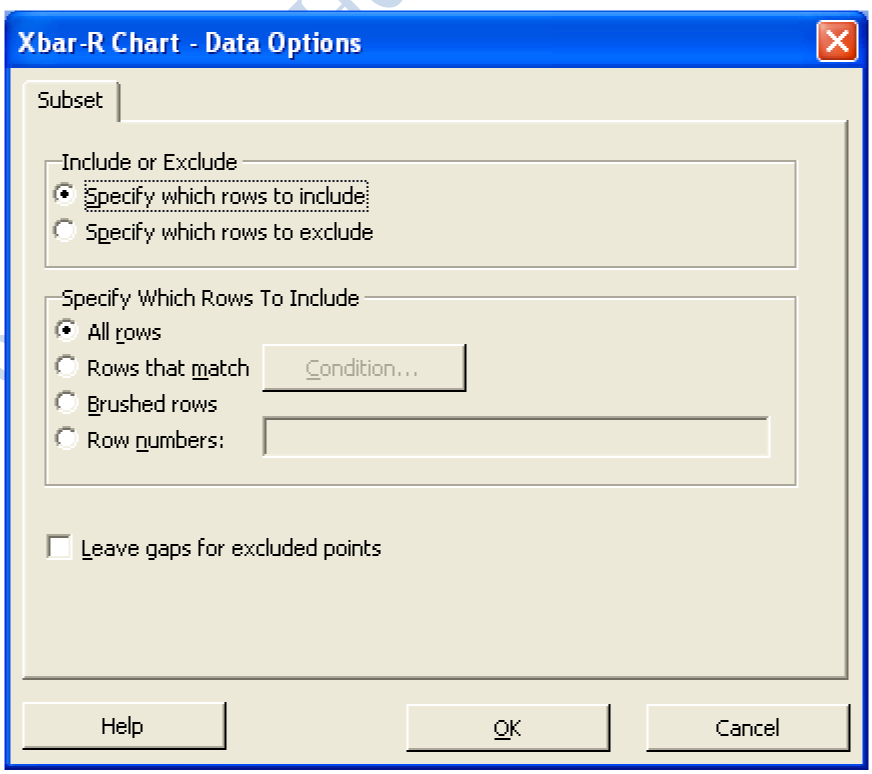


Figure A8.3
Minitab Xbar-R Chart Options Dialog Box, Estimate Tab

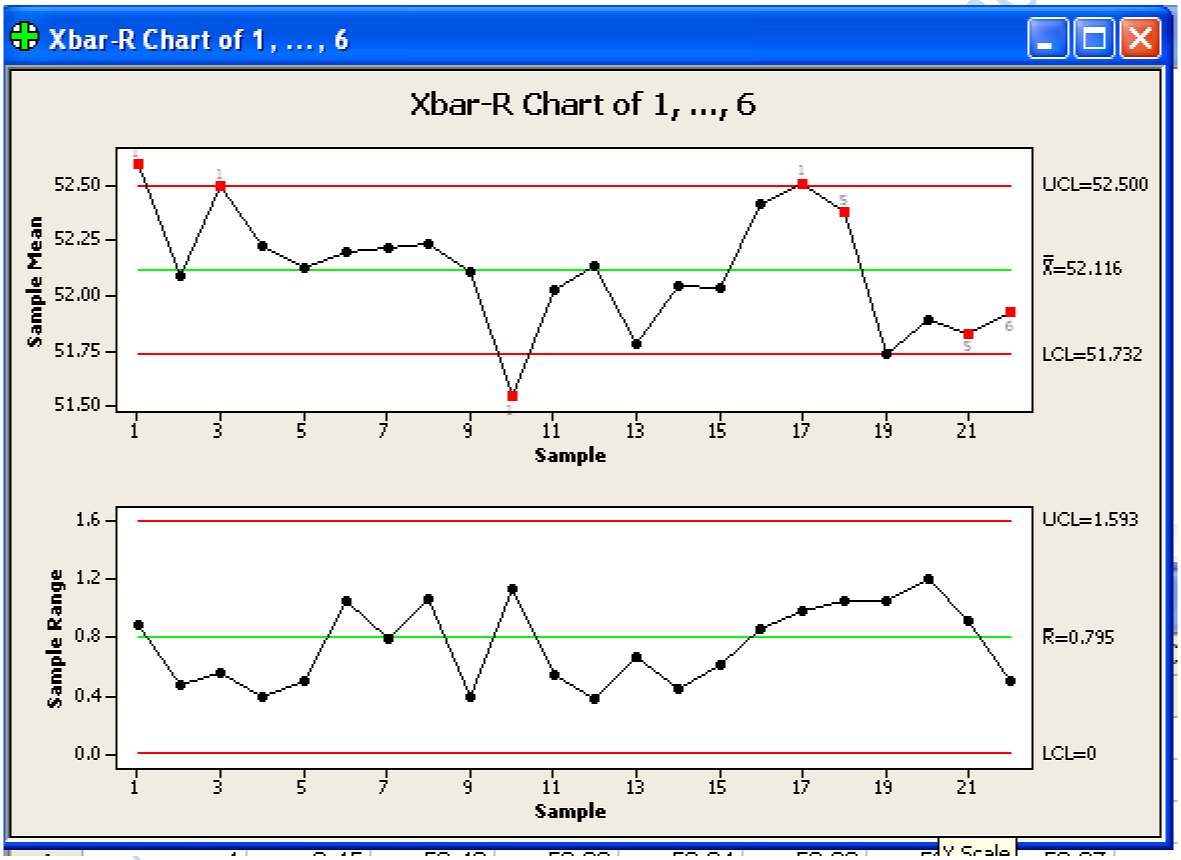


3. Note: When obtaining more than one set of $\bar{X}$ and R charts in the same session, be sure to reset the values of the points to be omitted before obtaining new charts.

4. In the Xbar-R Chart - Data Source Chart dialog box, click the **OK** button to obtain the *R* and $\bar{X}$ charts. The result can be found in Figure A8.4

Figure A8.4

Minitab Result for Vials



Using Minitab for the $\bar{X}$ and s Charts

$\bar{X}$ and s charts can be obtained from Minitab by selecting **Stat | Control Charts | Variable Charts for Subgroups | Xbar-S** from the menu bar. The format for entering the variable name is different, depending on whether the data are stacked down a single column or unstacked across a set of columns with the data for each time period in a single row. If the data for the variable of interest are stacked down a single column, choose Single Column in the Data are arranged as drop-down list box and enter the variable name in the edit box below. If the subgroups are unstacked with each row representing the data for a single time period, choose Subgroups as rows in the Data are arranged as drop-down list box and enter the variable names for the data in the edit box below.

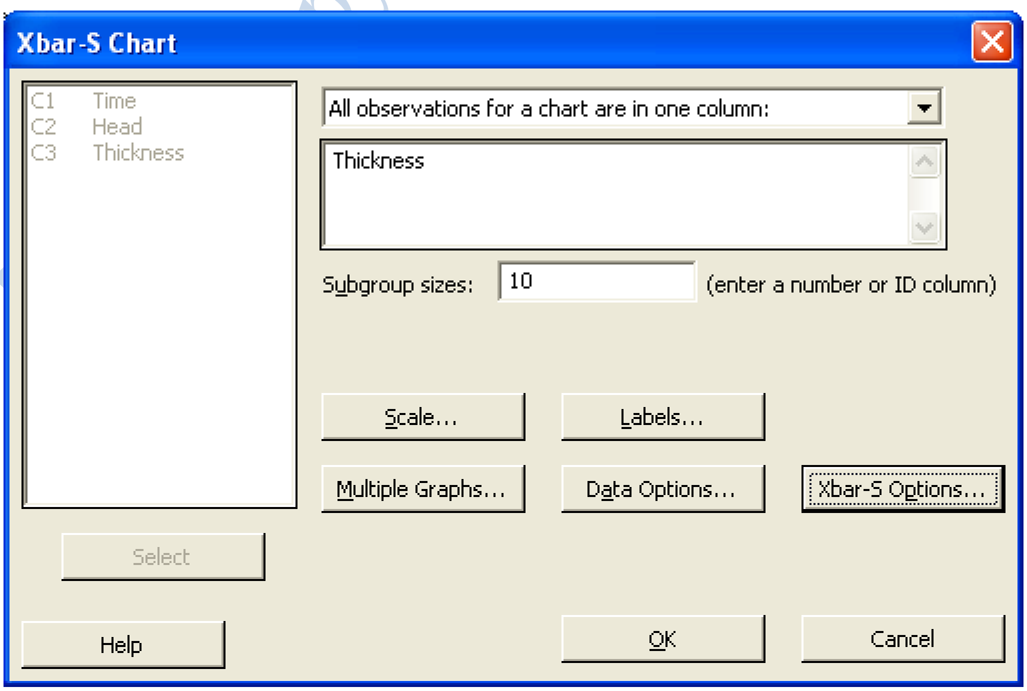
To illustrate how to obtain $\overline{X}$ and s charts, refer to the data of Table 8.4 concerning the coating thickness of plastic film. Open the **COATING.MTW** worksheet.

1. Select **Stat | Control Charts | Variable Charts for Subgroups | Xbar-S**. In the Xbar-S Chart - Data Source Chart dialog box, as shown in Figure A8.5, enter **Thickness** in the edit box. In the Subgroup sizes box, enter **10.** in the edit box. Click the **Xbar-S Options** button

2. In the Xbar-S Chart Options dialog box, click on the Tests tab. From the pull down menu select the **Perform all tests for Special Causes** option and enter the appropriate values on the right side for K. . Click the **OK** button to return to the Xbar-S Data Source Chart dialog box. These values will stay intact until Minitab is restarted.

3. If there are points that should be omitted when estimating the center line and control limits, click the **Data Options** tab in the Xbar-S Chart dialog box. Enter the rows to be omitted in the edit box shown. Click the **OK** button to return to the Xbar-S Chart dialog box.

Figure A8.5
Minitab Xbar-S Data Source Chart Dialog Box

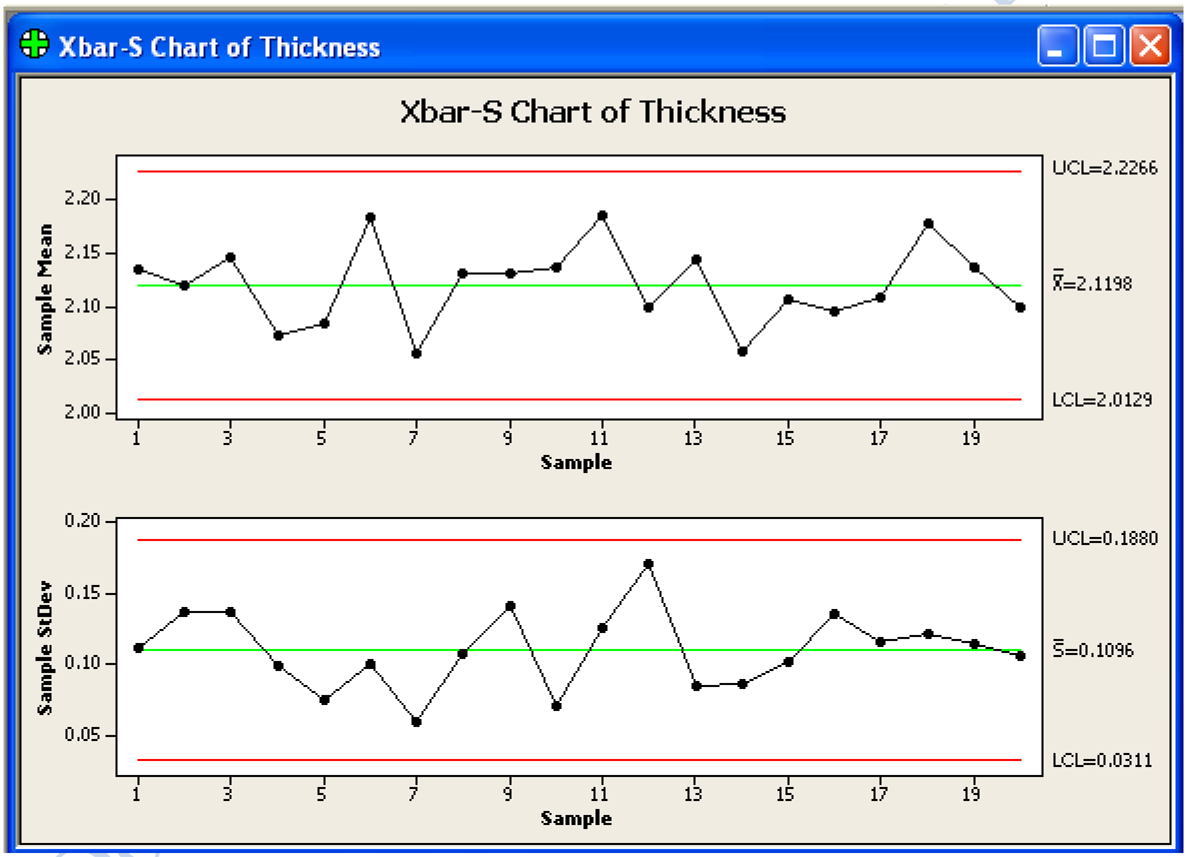


4. Note: When obtaining more than one set of *S* and $\overline{X}$ charts in the same session, be sure to reset the values of the points to be omitted before obtaining new charts.

5. In the Xbar-S Chart - Data Source Chart dialog box, click the **OK** button to obtain the *S* and $\overline{X}$ charts. The result it shown in Figure A8.6

Figure A8.6

Minitab Results for Coatings



Using Minitab for the Individual Value and Moving Range Charts

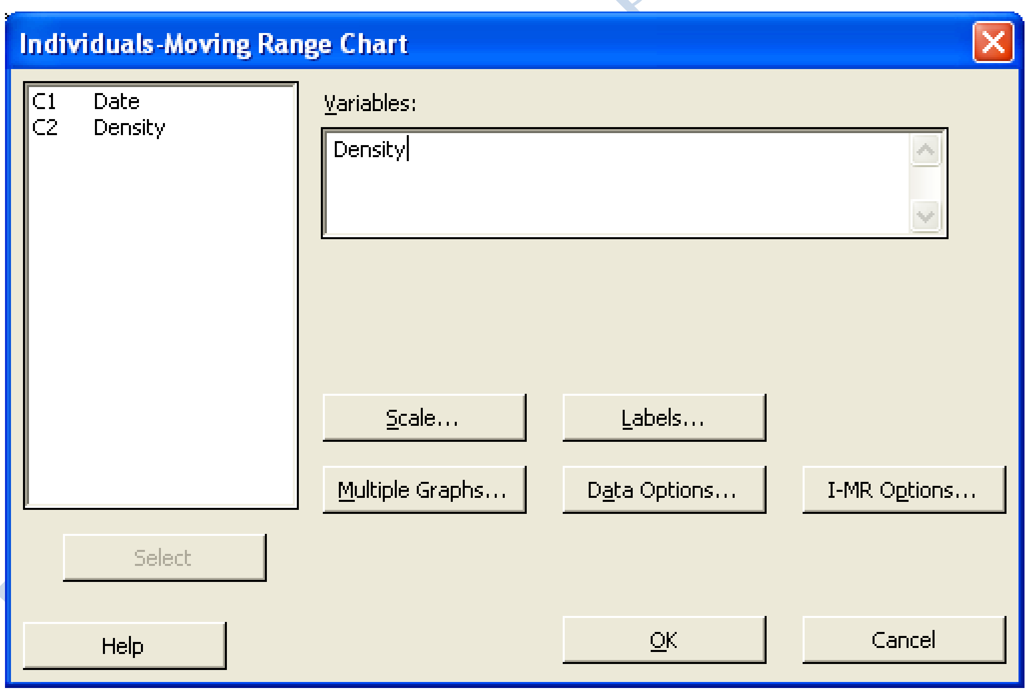
Individual Value and Moving Range charts can be obtained from Minitab by selecting **Stat | Control Charts | Variable Charts for Individuals | I-MR** from the menu bar. To illustrate how to obtain Individual Value and Moving Range charts, refer to the data of Table 8.5 concerning the weight of vials. Open the **DENSITY.MTW** worksheet.

1. Select **Stat | Control Charts | Variable Charts for Individuals | I-MR**. In the Individuals-Moving Range Chart - Data Source Chart dialog box (see Figure A8.7) enter **Density** in the variables edit box. Click the **I-MR Options** button.

2. In the I-MR Chart Options dialog box, click on the Tests tab. Select the **Tests** option button. The Minitab 16 default if **1 point > 3 standard deviations from center line**. Click the **OK** button to return to the Xbar-S Data Source Chart dialog box. These values will stay intact until Minitab is restarted.

3. If there are points that should be omitted when estimating the center line and control limits, click the **Data Options** tab in the I-MR Chart dialog box. Enter the rows of the points to be omitted in the edit box shown. Click the **OK** button to return to the I-MR Chart dialog box.

Figure A8.7
Minitab I-MR Data Source Chart Dialog Box



4. (Note: When obtaining more than one set of Individual Value and Moving Range charts in the same session, be sure to reset the values of the points to be omitted before obtaining new charts.)

5. In the I-MR Chart - Data Source Chart dialog box, click the **OK** button to obtain the individual value and moving range charts.

Using Minitab to Obtain Zone Limits

To plot zone limits on any of the control charts discussed in this appendix, open to the Data Source dialog box for the control chart being developed and do the following

1. Click the **Scale** button. Click the **Gridlines** tab. Select the **Y major ticks**, **Y minor ticks**, and **X major ticks** check boxes. Click the **OK** button to return to the Data Source dialog box.

2. Select the **Options** button. Select the **S limits** tab. In the Standard deviation limit positions: edit box, select **Constants** in the drop-down list box and enter **1 2 3** in the edit box. Click the **OK** button to return to the Data Source dialog box.