CHAPTER 7
ATTRIBUTE CONTROL CHARTS

Sections

Chapter Objectives

- To discuss when to use the different types of attribute control charts
- To construct the different types of attribute control charts: p chart, np chart, c chart and u chart
- To analyze and interpret attribute control charts
- To discuss the limitations of attribute control charts

7.1 Introduction

Attribute data are data based on classifying an item, such as a unit, a form, a person, an interaction into one of two categories, such as defective or non-defective, conforming or non-conforming, good or bad, etc., or counting the number of defects per item, such as number of errors per bank application, number of accidents per department and overall in a factory per month, etc. In Chapter 6, we saw that the first step on the ladder of quality consciousness is sorting defective from non-defective items. This step focuses on defect detection and on trying to inspect quality in by removing defective items. This stage is characterized by dependence on mass inspection, rather than statistical process control. Even today, many firms consider this quality control. Also in Chapter 6, we saw that the second step on the ladder of quality consciousness is improving a process to eliminate defectives or defects by using attribute statistical process control charts: p charts, np charts, c charts, or u charts, to be discussed in this chapter)

Information about why an item is either defective or contains defects (that is, fails to meet a given specification[s]) does not answer the question of why the specification was not met. Total (100 percent) conformance to specifications does not provide a mechanism for never-ending process improvement, or the reduction of unit-to-unit variation within specification limits. Reducing variation within specification limits, absent capital investment, results in higher quality outputs at lower cost. Chapter 8 discusses the third step on the ladder of quality consciousness: the continuous and never-ending reduction of unit-to-unit variation within specification limits through the use of variables statistical process control charts.

7.2 Types of Attribute Control Charts

There are two basic types of attribute control charts: **classification charts** and **count charts**. Each type is discussed below.

7.2.1 Classification Charts

Classification charts deal with the percentage of defective items in a subgroup of items. A subgroup defines a particular time period, such as a day, a week, a month, a quarter, or a year, place, such as location in a hospital, or a combination of time and place, such as percentage of errors in hospital Ward 5 South by month. They also deal with the number of items in a subgroup that have a particular characteristic, such as number of accidents, by department and overall, by month.

p Chart. The **p chart** is used to control the proportion, or percentage, of items with the characteristic of interest. Subgroup sizes in a p chart may remain constant or may vary. A p chart might be used to control defective versus non-defective items, or acceptable versus not acceptable items.

np Chart. The **np chart** serves the same function as the p chart except that it is used to control the number rather than the fraction of items with the characteristic of interest. It is only used with constant subgroup sizes. It delivers the same information as a p chart.

7.2.2 Count Charts

Count charts deal with the number of times a set of characteristics, such as defects, appear in some given **area of opportunity**. A defect can be an omission of a piece of information on a bank form, an accident in a hospital, or a warranty claim for a particular model car, for example. An area of opportunity can be a bank form, a geographical area in which one or more accidents can occur in a hospital, or a time period in which warranty claims are made.

*c Chart***.** A **c chart** is used to control the number of times a particular characteristic, such as defects, appears in a constant area of opportunity. A constant area of opportunity is one in which each subgroup used in constructing the control chart provides the same area or number of places in which the characteristic of interest may occur. For example, defects per air conditioner, accidents per workweek in a factory, and deaths per week in a city all provide an approximately constant area of opportunity for the characteristic of interest to occur. The area of opportunity is a subgroup of constant size, whether it is the air conditioner, the factory workweek, or the week in the city.

u Chart. A **u chart** serves the same basic function as a c chart, but it is used when the area of opportunity changes from subgroup to subgroup. For example, we may examine varying square footage of paper selected from rolls of paper for blemishes, or carloads of lumber for damage when the contents of the rail cars vary from rail car to rail car.

7.2.3 Manual Construction of Attribute Control Charts

Whether data is collected manually or electronically, standard forms exist for the construction of attribute control charts. Although there may be some slight individualizing from firm to firm, certain standard areas are almost always provided on the forms. Figure 7.1 shows an example of an attribute control chart form.

In the upper left corner the plant/factory/office location is entered; then just to the right the type of control chart is noted. The next box requires information on the part name and number. Other identifying entries include the department and the operation number and name. The next two boxes provide space to enter the process average, UCL, and LCL plus the date on which they were calculated.

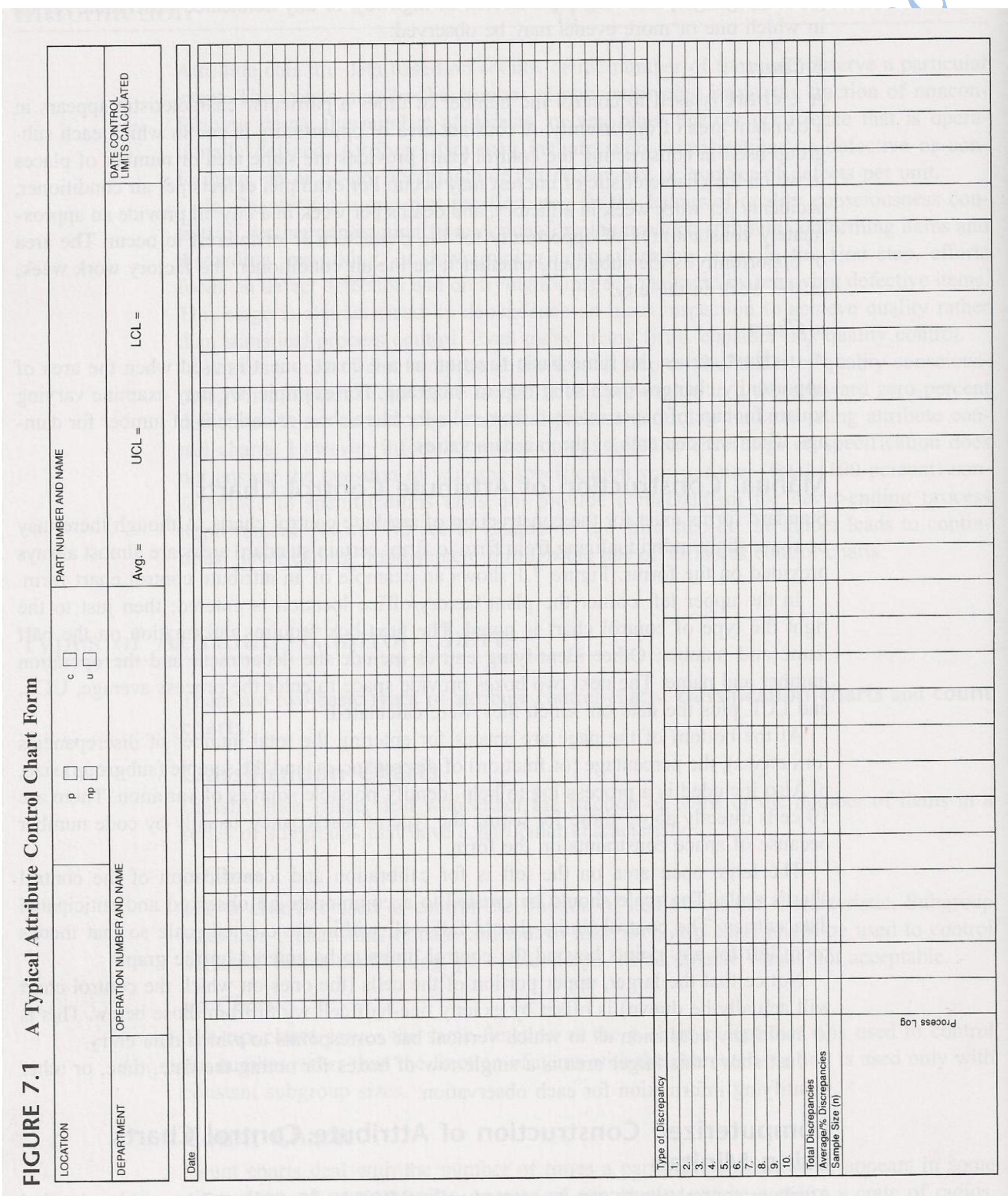
At the bottom of the page are spaces for entering the total number of discrepancies, or defects; the percentage, or fraction, of discrepancies; and the sample, or subgroup, size, n. Also included is a process log sheet (a diary of defects and circumstances that may affect the process) to help identify possible sources of variation. There are ten cells directly above these for listing the type of discrepancy, usually by code number because of space constraints on the form.

The large open area on the left is for calibration and identification of the control chart's scale. The scale should be created to accommodate all observed and anticipated data entries. The control limits should fall well within the created scale, leaving room left for any points beyond the control limits to be entered on the graph.

Notice that the larger, upper portion of the cells (the ones on which the control chart will actually be drawn) is offset by exactly one half-cell width from those below. This is to avoid any confusion as to which vertical bar corresponds to which data entry.

Just above this larger area is a single row of boxes for noting the date, time, or other identifying information for each observation.

Figure 7.1
A Typical Attribute Control Chart Form



7.3.2 When Not to Use p Charts or np Charts

Occasionally data based on measurements (variables data) are downgraded into data in terms of conformance or nonconformance (attribute data). This is not a good practice because the data based on measurements can provide more information than the data based on conformance or nonconformance.

It is also important that the denominator in the fraction being charted is the proper area of opportunity. If it is not, then the data are not truly a proportion but a ratio. For example, the fraction of defectives found on the second shift will be a useful proportion only if it is computed by dividing the number of defectives found on the second shift by the proper area of opportunity: the number of units produced on the second shift. If a ratio is created using the number of defectives found on the second shift divided by the number of items shipped by the second shift, there is no way of knowing that the items shipped during the second shift were all produced on the second shift. Some items shipped on the second shift may have been produced during the first shift and therefore this may be an inappropriate area of opportunity.

Last, we must exercise caution to ensure that the control chart is being created for a single process. Control charting output from combined different processes will result in irrational subgroups that will not enable us to distinguish special from common causes of variation. Little if anything can be learned from such charts and the net effect may be a masking of special causes of variation.

7.3.3 Constructing Classification Charts

An adaptation of the **Deming Cycle**, discussed in Chapter 2, may be used to construct and interpret a p chart or np chart.

I. Plan

a. The process to be studied using the control chart must be named and flowcharted.
b. The purpose of the chart must be determined.
c. The characteristic to be charted must be selected and operationally defined.
d. The manner, size, and frequency of subgroup selection must be established.
e. The type of chart (i.e., p chart or np chart) must be established.
f. Forms for recording and constructing the control chart must be established.

II. Do

a. Data must be recorded either manually onto control chart paper or electronically onto an Excel or Minitab worksheet; see Appendix 7 for instructions on using Minitab to create attributes control charts.
b. The fraction of items with the characteristic of interest must be calculated for each of the subgroups, either manually or electronically by Minitab.
c. The average value must be calculated, either manually or electronically by Minitab.

d. The control limits and zone boundaries must be calculated and plotted onto the control chart, either manually or electronically by Minitab.
e. The data points must be entered on the control chart, either manually or electronically by Minitab.

III. Study
a. The control chart must be examined for indications of special causes of variation, either manually or electronically using the Test option in Minitab.
b. All aspects of the control chart must be reviewed periodically and appropriate changes made when required.

IV. Act
a. Actions must be undertaken to bring the process under control by eliminating any negative special causes of variation, or instilling any positive special causes of variation.
b. Actions must be undertaken to reduce the causes of common variation for the purpose of never-ending improvement of the process.
c. Specifications must be reviewed in relation to the capability of the process.
d. The purpose of the control chart must be reconsidered by returning to the Plan stage.

The Plan Stage. The first step in the Plan stage is to name and flowchart the process to be studied using the control chart.

The second step in the Plan stage is to determine the purpose of the chart.

1. For data at the process level, a p chart or np chart may be created to search for special causes of variation in a chaotic system, or to search for the common causes of variation in a stable system.
2. For data that has been aggregated over two or more processes, a p chart or np chart may be used to keep management from over-reacting to common causes of variation.

The third step in the Plan stage is to select and operationally define the characteristic for control charting. Very often a single item possesses several characteristics, any of which may cause the item to be considered defective or nonconforming. Generally, a single chart will be kept for the entire item, but frequently separate charts will be kept for individual characteristics. It is usually efficient to concentrate initial efforts on control charts for the characteristics that cause problems for the customer and are within control of the process owner studying the problem. Some of the techniques to be discussed in Chapter 10, such as brainstorming, may be useful in selecting the characteristics to be charted.

The fourth step in the Plan stage is to determine the manner, size, and frequency for the selection of subgroups. The **subgroup size** is the number of items to be observed at each sampling to determine the fraction conforming or nonconforming. As we will see in Chapter 8, **rational subgroups** should be selected to minimize within-subgroup variation. Frequently subgroups are selected in the order of production or over time. The decisions concerning the method of selection and the factors to be isolated will require careful planning by those individuals with knowledge of, and experience with, the process. Early efforts may need revision as a result of unexpected factors that may be revealed while developing the control chart. This may lead to the creation of several charts where only one was initially contemplated, but this may be of use in resolving special causes of variation and reducing common variation in the areas charted.

The necessary subgroup size will be discussed later in this chapter, in section 7.4.4.

The frequency with which the subgroups are selected is generally specific to each situation and depends upon factors such as the rate of production, elapsed time, and shift duration. The frequency should be logical in terms of shifts, time periods, or any other rational grouping. The shorter the intervals between subgroups, the more quickly information may be fed back for possible action. Cost will naturally be a factor, but after process stability has been established, frequency of subgroup selection can often be decreased and efforts focused elsewhere.

The fifth step in the Plan stage is to decide whether to use a p chart or an np chart. There is no substantive difference between these two charts. The information portrayed is essentially the same; only the form is different. The p chart displays the *fraction* with the characteristic of interest, while the np chart displays the *number* of items with that characteristic of interest. From a technical standpoint, they may be used interchangeably. Nevertheless, as the np chart permits the data to be entered as whole numbers (rather than as the ratio of the number of nonconforming items to the subgroup size), the np chart may be preferable. However, as we will discuss later in this chapter, if subgroup size varies from subgroup to subgroup, a p chart is typically used.

The final step in the Plan stage is to select the control chart form. Standard forms are available from the American Society for Quality Control for attribute control charts. [American Society for Quality] Many firms have developed their own forms, such as the one in Figure 7.1. Alternatively, Minitab may be used to construct the control chart form, as described in Appendix A7.1.

Occasionally, supplemental forms, or **check sheets**, are used to collect the initial data, as shown in Figure 7.2. The data are then transferred to a control chart. This technique may be especially convenient if the control chart is to be drawn at another time or with the aid of a computer or if the work environment is not suitable for drawing the chart.

Figure 7.2
Sample Data Collection Form for p Charts and np Charts

Data Sheet for a p-chart or an np-chart						
Department:						
Part Name:				**Part Number:**		
Date	**Time**	**Inspected By**	**Number Inspected**	**Number Defective**	**Fraction Defective**	**Comments**

*The Do Stage***.** The Do stage begins with the recording of the required data for each subgroup on either the data collection sheet, directly on the control chart paper, or onto a Minitab worksheet. Any abnormalities or unusual occurrences should be recorded in the space provided for comments on the control chart form, or on a special **log sheet**. Log sheets are diaries that record historical data by subgroup and are used to provide clues to special causes of variation, should a lack of control be found. Hence, they are critical to the proper use of a control chart that is constructed for data at the process level. Recall that this type of control chart is created to search for special causes of variation in a chaotic system, or search for the common causes of variation in a stable system.

If the chart is a p chart, the fraction of items with the characteristic of interest must be calculated for each subgroup, either manually or by Minitab. After the data for each subgroup have been collected (using at least 20 subgroups), the average value for p is calculated using Equation (6.1), either manually or using Minitab. This value provides a centerline for the control chart and is the basis for the calculation of the standard error used to determine the control limits and zone boundaries.

Next, the control limits and zone boundaries are computed -- using the equations introduced in Chapter 6 and discussed later in this chapter -- and are then drawn onto the control chart, either manually or using Minitab.

Last in the Do stage, the p values (or np values for the np chart) are plotted onto the control chart, either manually or using Minitab.

It is usually desirable to complete the control chart promptly and display it for those individuals working with the process. It is not unusual for such a display to have immediate beneficial results, especially if those involved have been educated about the purpose and meaning of control charts.

*The Study Stage***.** Using the rules introduced in Chapter 6 and detailed in Chapter 9, we either manually examine the control chart from right to left (looking backward in time) for indications of a lack of control, or use Minitab to examine the data for a lack of control. Any special causes are appropriately noted on the control chart.

Periodically the centerline, control limits, and zone boundaries should be reviewed. Timing of the review, of course, depends on the process and its history. Typically p charts and np charts are kept for long periods of time. Any change in the process is cause to consider a review of chart parameters.

*The Act Stage***.** Indications of special sources of variation may be revealed during the Study stage. If any special sources of variation are found, steps must be initiated to remove the sources if they are bad, or incorporate them into the process if they are good. This is accomplished by creating a revised flowchart of the process that utilizes the modifications required to resolve any special causes of variation. It is not uncommon for supervisors or foremen to already be aware of problem areas; the control chart helps to discover the cause and reinforce arguments for improvement. Furthermore, control charts help focus attention on areas needing immediate help.

If it appears there is a lack of control on the desirable side of the chart, it is a good practice to examine the inspection procedures; faulty inspection procedures may be to blame. On the other hand, perhaps a special cause is responsible for points on the desirable side of the chart that should be formally incorporated into the process -- that is, improvements may have spontaneously occurred in the process that, once discovered, should be incorporated.

Although a control chart may reveal no indications of special causes of variation, the overall level of the fraction or number of items with the characteristic of interest may not be at a satisfactory level (the **threshold state**). Other tools and techniques, such as Cause and Effect diagrams and Pareto analysis, to be discussed in Chapter 10, may be used in an attempt to reduce the high fraction of nonconforming items as the PDSA cycle rolls as a wheel up the hill of never-ending process improvement.

In the drive toward never-ending improvement, no level of defectives is low enough. Nevertheless, as the proportion of defectives shrinks as a result of efforts at process improvement, then subgroups will often contain no defectives. This will make the use of p charts or np charts difficult to use because of the large subgroup sizes needed to detect even a single defective item. This leads to the use of variables control charts, which we discuss in Chapter 8.

Last in the Act stage is the reconsideration of the purpose of the control chart. We return to the beginning of the Plan stage, where the cycle begins again.

7.4 The p Chart for Constant Subgroup Sizes

In Chapter 6 we saw an example of a p chart with a constant subgroup size. Constant subgroup size implies that the same number of items is sampled and then classified for each subgroup on the chart. We use a discrete, countable characteristic of output to construct a p chart; for example, the fraction of customers who pay their bills in fewer than 30 days, the fraction of correspondence sent electronically, or the fraction of an airline's flights that arrive within 15 minutes of their scheduled arrival time.

7.4.1 The Centerline and Control Limits

For a stable process, categorization of data into two classes suggests that every item has approximately the same probability of being in one of the two categories. We say approximately because even stable processes exhibit variation.

The centerline for the p chart is established at $\bar{p}$, the overall fraction of output that was nonconforming, as given by Equation (7.1). The upper control limit and the lower control limit are found by adding and subtracting three times the standard deviation of the faction defective (given by Equation (7.2)) from the centerline value using Equations (7.3) and (7.4).

$$\text{Centerline(p)} = \bar{p} = \left[\frac{\text{Total number of defectives in all subgroups under investigation}}{\text{Total number of units examined in all subgroups under investigation}}\right] \qquad (7.1)$$

$$\text{UCL(p)} = p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (7.2)$$

$$\text{LCL(p)} = p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (7.3)$$

7.4.2 Construction of a p chart: An Example

As an illustration, consider the case of an importer of decorative ceramic tiles. Some tiles are cracked or broken before or during transit, rendering them useless scrap. The fraction of cracked or broken tiles is naturally of concern to the firm. Each day a sample of 100 tiles is drawn from the total of all tiles received from each tile vendor. Table 7.1 presents the sample results for 30 days of incoming shipments for a particular vendor.

Table 7.1

Daily Cracked Tiles

Day	Sample Size	Number Cracked	Fraction
1	100	14	0.14
2	100	2	0.02
3	100	11	0.11
4	100	4	0.04
5	100	9	0.09
6	100	7	0.07
7	100	4	0.04
8	100	6	0.06
9	100	3	0.03
10	100	2	0.02
11	100	3	0.03
12	100	8	0.08
13	100	4	0.04
14	100	15	0.15
15	100	5	0.05
16	100	3	0.03
17	100	8	0.08
18	100	4	0.04
19	100	2	0.02
20	100	5	0.05
21	100	5	0.05
22	100	7	0.07
23	100	9	0.09
24	100	1	0.01
25	100	3	0.03
26	100	12	0.12
27	100	9	0.09
28	100	3	0.03
29	100	6	0.06
30	100	9	0.09
Totals	**3,000**	**183**	

The average fraction of cracked or broken tiles can be calculated from this data using Equation (7.1). This is the centerline for the control chart.

$$\text{Centerline}(p) = \bar{p} = \frac{183}{3000} = 0.061$$

The upper and lower control limits can then be computed using Equations (7.2) and (7.3).

$$UCL(p) = 0.061 + 3\sqrt{\frac{0.061(1-0.061)}{100}} = 0.133$$

$$LCL(p) = 0.061 - 3\sqrt{\frac{0.061(1-0.061)}{100}} = -0.011$$

Recall from our discussion in Chapter 6 that a negative lower control limit in a p chart is meaningless; instead we use a value of 0 for the lower control limit.

For a stable process, the probability that any subgroup fraction will be outside the three-sigma limits is small (approximately 1,350 per million above the upper control limit and the same probability below the lower control limit). Also, if the process is stable, the probability is small that the data will demonstrate any other indications of the presence of special causes of variation by virtue of the other rules discussed in Chapters 6 and 9. But if the process is not in a state of statistical control (i.e., it exhibits one or more special causes of variation), the control chart provides an economical basis upon which to search for and identify indications of this lack of control. Additionally, for p charts, the six other rules for out-of-control points described in Chapter 6 can all be applied. In order to do so, we need to compute the boundaries for the A, B, and C zones.

Recall from Chapter 6 that the width of each zone is one standard error, or one third of the distance between the upper control limit and the centerline. Thus the boundaries between zones B and C are one standard error on either side of the centerline. Here they are found by adding and subtracting the quantity $\sqrt{\bar{p}(1-\bar{p})/n}$ from the centerline, $\bar{p}$.

$$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{0.061(1-0.061)}{n}} = 0.024$$

So that:

Boundary between upper zones B and C

$$= \bar{p} + \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (7.4)$$

In our example this value is 0.061 + 0.024 = 0.085 and

Boundary between lower zones B and C

$$= \bar{p} - \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (7.5)$$

In our example this value is 0.061 - 0.024 = 0.037.

We find the upper and lower boundaries between zones A and B by adding and subtracting, respectively, two standard errors from the centerline, $\bar{p}$.

Boundary between upper zones A and B = $\bar{p} + 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ (7.6)

and

Boundary between lower zones A and B = $\bar{p} - 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ (7.7)

Using these in our example,

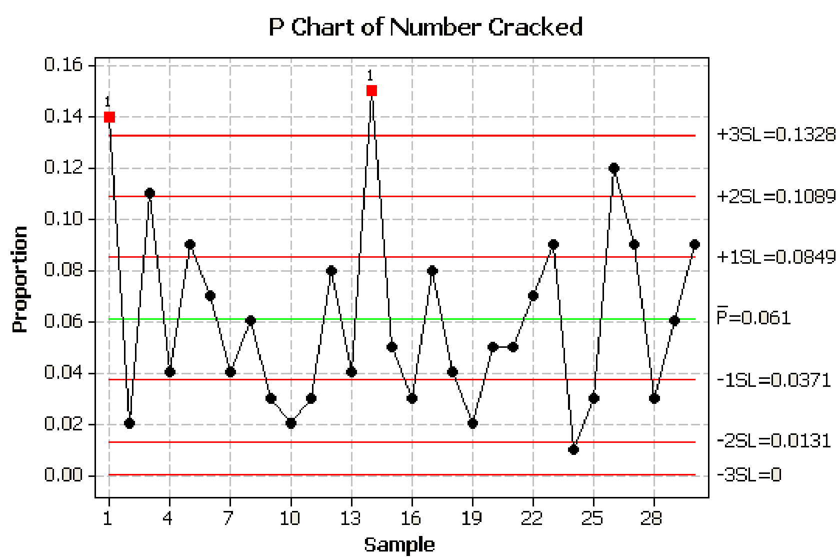
0.061 + 2(0.024) = 0.109

and

0.061 – 2(0.024) = 0.013

Figure 7.3 shows the completed control chart.

Figure 7.3
p chart for Fraction of Cracked Tiles



Examining the chart, we find a process that is out of control. On day 1, the average fraction of defective tiles is above the upper control limit. The average fraction of defective tiles for day 14 is also above the upper control limit, another indication of lack of control. None of the other rules presented in Chapter 6 appears to be violated. That is, there are no instances when two out of three consecutive points lie in zone A on one side of the centerline; there are no instances when four out of five consecutive points lie in zone B or beyond on one side of the centerline; there are no instances when eight consecutive points move upward or downward; nor are there eight consecutive points on one side of the centerline. There does not appear to be a lack of runs; there are no instances of 13 consecutive points in zone C.

Nevertheless, the incoming flow of ceramic tiles needs further examination. The special causes of these two erratic shifts in the fraction of cracked or broken tiles should be eliminated so that expectations for usable portions can be stabilized. Only after this is done can improvements be made in the process.

Further study reveals that on both day 1 and day 14 the regular delivery truck operator was absent because of illness. Another employee loaded and drove the delivery truck on those days. That individual had never been instructed in the proper care of the product, which requires special handling and treatment. To solve this problem and eliminate this special cause of variation, management created and implemented a training program using the regular driver's experience for three other employees. Any one of these three employees can now properly fill in and perform satisfactorily. Thus the system has been changed to eliminate this special cause of variation.

After the process has been changed so that special causes of variation have been removed, the out-of-control points are removed from the data. The points are removed from the control chart, and the graph merely skips over them.

Removing these points also changes the process average and standard error. Therefore the centerline, control limits, and zone boundaries must be recalculated.

The new centerline and control limits are:

$$\bar{p} = 154/2800 = 0.055$$

$$\text{UCL}(p) = 0.055 + 3\sqrt{\frac{(0.055)(0.945)}{100}} = 0.123$$

$$\text{LCL}(p) = 0.055 - 3\sqrt{\frac{(0.055)(0.945)}{100}} = -0.013\,(0.000)$$

The new upper and lower boundaries between zones B and C are calculated using Equations (7.4) and (7.5):

Boundary between upper zones B and C = $0.055+\sqrt{\frac{(0.055)(0.945)}{100}}=0.078$

Boundary between lower zones B and C = $0.055-\sqrt{\frac{(0.055)(0.945)}{100}}=0.032$

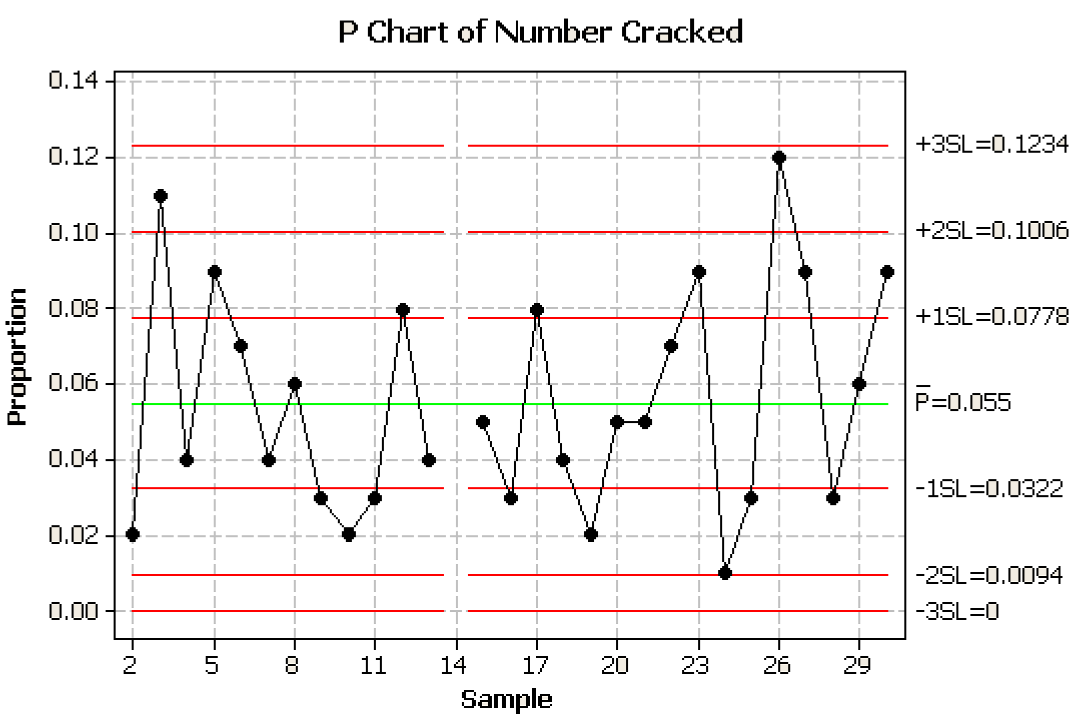
The new upper and lower boundaries between zones A and B are calculated using Equations (7.6) and (7.7):

Boundary between upper zones A and B = $0.055+2\sqrt{\frac{(0.055)(0.945)}{100}}=0.101$

Boundary between lower zones A and B = $0.055-2\sqrt{\frac{(0.055)(0.945)}{100}}=0.009$

The entire control chart is redrawn, as shown in Figure 7.4. None of the seven rules discussed in Chapter 6 is violated, so there does not appear to be a lack of control. The process now appears to be stable and in a state of statistical control. Management may now look for ways to reduce the overall process average of the number of cracked or broken tiles to raise the usable number of tiles per shipment and effectively increase the process output.

Figure 7.4
Revised p Chart for Fraction of Cracked Tiles



7.4.3 Iterative Reevaluations

It is possible -- and not at all uncommon -- that by changing the process, removing points that were out of control, and recomputing the control limits and zone boundaries, points that initially exhibited only common variation will now indicate a lack of control. If and when this happens, the system must again be reevaluated to eliminate the newly revealed special causes of variation.

This may once again uncover even more indications of a lack of control, which also must be removed from the system. Analysis of the process will continue to iterate in this manner until there no longer appears to be a lack of control. Keep in mind that in the course of these iterations, some of the data will be discarded. Hence the data base will shrink, and the control chart will be based on fewer and fewer subgroups. Furthermore, as changes are made, the process may no longer resemble the original process.

We must also keep in mind that if control limits are recalculated too frequently (as might be the temptation with automatic data processing available with many computer control routines), it becomes possible to mistake common variation for special variation. This effect parallels the over-steering many new drivers experience when first learning to drive a car. Knowledge and experience with the process are the best guides here. As a general rule, control chart statistics are recomputed whenever there is a change to the process.

At some point a decision must be made to stop analyzing the original data and collect new data. There is no explicit rule for the point at which this should be done; only knowledge and experience with the process can dictate when to stop analyzing previous data and begin collecting and analyzing new data.

7.4.4 Subgroup Size

When constructing a p chart, the subgroup size is much larger than that required for variables control charts. This is because the sample size must be large enough that some nonconforming items are likely to be included in the subgroup. If, for example, a process produces 1.0 percent defectives, sample subgroups of size 10 will only occasionally contain a nonconforming item. As a general rule of thumb, control charts based on classification count data should have sample sizes large enough so that the average count per subgroup is at least 2.00. This allows the A, B, and C zones to be wide enough to provide a reasonable working region into which data points may fall for analysis. This is true for both the p chart and the np chart, which we discuss later in this chapter.

Consider, for example, a process producing 5 percent of its output with a particular characteristic of interest (i.e., $\bar{p} = 0.05$). Subgroups of size 20 yield an average count of only 20(0.05) = 1.0. Further, each subgroup would have an integer number of counts, yielding fractions in increments of 5 percent. The centerline would be at 0.050, the lower control limit would be at 0.000, and the upper control limit would be at

$$0.05+3\sqrt{\frac{(0.05)(1-0.05)}{20}}=0.196$$

Only fractions of 0.00, 0.05, 0.10 and 0.15 would fall within the control limits. Examining patterns such as runs up or down would not be practical; finding eight points moving upward or downward would almost always be redundant because the beginning or end of the run would be beyond the upper control limit and would indicate a lack of control for that reason. Clearly we would not be able to learn too much from a p chart based on a subgroup size of 20 items with its centerline at 0.05. Similarly, samples taken from a process producing nonconforming items at a rate of only 1 percent would require samples of 200 to have an average count of 2.00. Even with samples of size 200, samples would provide counts of 0, 1, 2, and so on for subgroup fractions in increments of 0.005. With a centerline at 0.01, the p chart would not be very detailed and might not provide satisfactory indications of a lack of control.

Average subgroup counts of fewer than 2.00 present problems that can become extreme, especially if the average count per subgroup falls below 1.00. Hence, subgroups must be made large enough so that the average count is at least 2.00.

Ideally, subgroup sizes should remain the same for all subgroups, but occasionally circumstances require variations in subgroup size. Whether the subgroup size for a p chart varies or remains constant, the larger the subgroup size, the narrower the control limits will be. This is because the subgroup size, n, appears in the denominator of the expression for the standard error; the larger the value for n, the narrower the width of the control limits and zones A, B, and C around the process average will be.

7.4.5 Subgroup Frequency

Every process goes through physical cycles, such as shifts and ordering sequences. p chart and np chart calculations must be based on a sufficient number of subgroups to encompass all of the cycles of a process to include all possible sources of variation. Subgroup data should be collected at a frequency greater than the frequency at which the process can change. This frequency is determined by a process expert.

7.4.6 Number of Subgroups

As a rule of thumb, the number of subgroups should be at least 25 for p charts and np charts.

7.4.7 Subgroups Not Based on Time

It is possible to construct control charts for rational subgroups that do not represent chronological events. For example, a p chart for the fraction defective produced by a battery of 100 machines performing the same task (such as spot welding) might be kept on a single control chart for a given month. In these situations, the number of subgroups must encompass all machines to encompass all possible sources of variation. Additionally, the rules concerning indications of a lack of control by virtue of trends over time in the data -- such as two out of three consecutive points in zone A or four out of five consecutive points in zone B or beyond -- should be ignored.

7.4.8 Construction of a p Chart: Another Example

An injection molding process provides a bracket to be used on aircraft passenger seats. Daily samples of 500 brackets are selected from the production output and examined carefully for cracks, splits, or other imperfections that will render them defective. Table 7.2 lists the results.

Table 7.2

Defective Aircraft Seat Brackets

Day	Sample Size	Number of Defectives	Fraction Defective
1	500	12	0.024
2	500	9	0.018
3	500	8	0.016
4	500	10	0.02
5	500	17	0.034
6	500	33	0.066
7	500	15	0.03
8	500	46	0.092
9	500	22	0.044
10	500	13	0.026
11	500	9	0.018
12	500	15	0.03
13	500	4	0.008
14	500	37	0.074
15	500	20	0.04
16	500	15	0.03
17	500	14	0.028
18	500	18	0.036
19	500	45	0.09
20	500	25	0.05
21	500	27	0.054
22	500	33	0.066
23	500	17	0.034
24	500	28	0.056
25	500	12	0.024
Totals	**12,500**	**504**	

The centerline, control limits, and zone boundaries are calculated from Equations (7.1) through (7.7):

Centerline (p) = $\bar{p}$ = 504/12500 = 0.040

$$UCL(p) = 0.040 + 3\sqrt{\frac{(0.040)(0.960)}{500}} = 0.066$$

$$LCL(p) = 0.040 - 3\sqrt{\frac{(0.040)(0.960)}{500}} = 0.014$$

$$\text{Boundary between lower zones A and B} = 0.040 - 2\sqrt{\frac{(0.040)(0.960)}{500}} = 0.022$$

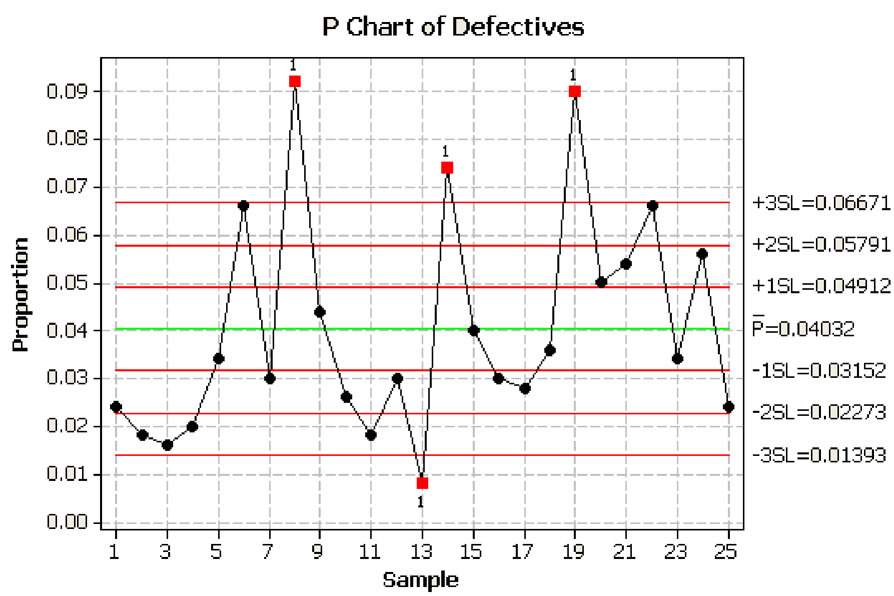
$$\text{Boundary between lower zones B and C} = 0.040 - \sqrt{\frac{(0.040)(0.960)}{500}} = 0.031$$

$$\text{Boundary between upper zones B and C} = 0.040 + \sqrt{\frac{(0.040)(0.960)}{500}} = 0.049$$

$$\text{Boundary between upper zones A and B} = 0.040 + 2\sqrt{\frac{(0.040)(0.960)}{500}} = 0.058$$

Figure 7.5 illustrates the p chart for these data. Many points indicate that this process was not in a state of statistical control. The operator running the molding process initiates a study that reveals that the mold is poorly designed, so consistent parts cannot be fabricated. The operator suggests a redesign of the mold that may eliminate most of the special causes of variation and reduce the average fraction defective. Once the new mold is put into use, the operators can collect more data and make a new p chart to determine if the proportion of defectives is stable over time, and if the average proportion defective has been reduced by the new mold.

Figure 7.5
p chart for Fraction of Defective Aircraft Seat Brackets



7.5 The p Chart for Variable Subgroup Sizes

Sometimes subgroups vary in size. This makes the manual construction of a p chart somewhat more tedious. The standard error, $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, varies inversely with the sample size. That is, as the sample size increases, the standard error decreases, and vice versa. Control limits and zone boundaries are calculated based on the standard error. Consequently, as the sample size changes so will the control limits and the zone boundaries.

7.5.1 Using Varying Control Limits: An Example

When sample sizes vary from subgroup to subgroup, we calculate new zone boundaries and control limits for each subgroup.

Consider, for example, the case of a highway toll barrier with two types of toll collection mechanisms: automatic and manned. The automatic lanes require exact change or a transponder while the manned lanes do not. The fraction of vehicles arriving with exact change or a transponder is examined using a control chart for a series of rush hour intervals on consecutive weekdays. As the number of vehicles passing through the toll barrier varies from day to day, the control limits change day to day. One-hour periods (7:30 to 8:30 am) for 20 consecutive weekdays yield the data in Table 7.3.

Table 7.3

Number of Vehicles Using Exact Change

Day	n	Number with Exact Change	Day	n	Number with Exact Change
1	465	180	11	406	186
2	123	38	12	415	149
3	309	142	13	379	90
4	83	20	14	341	148
5	116	35	15	258	107
6	306	108	16	270	84
7	333	190	17	480	185
8	265	106	18	350	184
9	354	94	19	433	210
10	256	116	20	479	197
			Totals	**6,421**	**2,569**

Using these data, $\bar{p}$, the centerline, can be calculated from Equation (7.1) as

$$\text{Centerline(p)} = \bar{p} = 2569/6421 = 0.400$$

We can also calculate each UCL, LCL, and zone boundary using Equations (7.2) through (7.7).

For example, for the first data point,

$$UCL(p) = 0.40 + 3\sqrt{\frac{(0.40)(1-0.40)}{465}} = 0.468$$

$$LCL(p) = 0.40 - 3\sqrt{\frac{(0.40)(1-0.40)}{465}} = 0.332$$

Boundary between lower zones A and B = $0.40 - 2\sqrt{\frac{(0.040)(1-0.40)}{465}} = 0.355$

Boundary between lower zones B and C = $0.40 - \sqrt{\frac{(0.40)(1-0.40)}{465}} = 0.377$

Boundary between upper zones A and B = $0.40 + 2\sqrt{\frac{(0.40)(1-0.40)}{465}} = 0.423$

Boundary between upper zones B and C = $0.40 + \sqrt{\frac{(0.40)(1-0.40)}{465}} = 0.423$

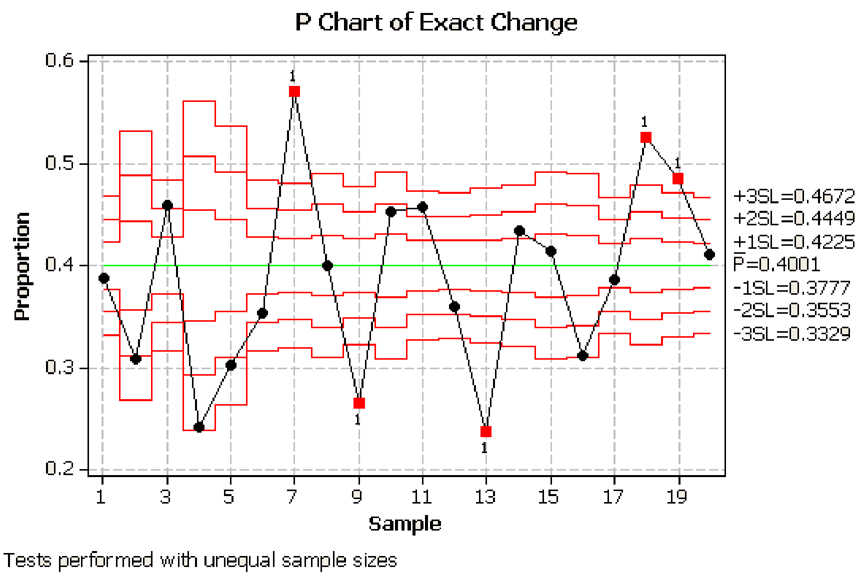
Since subgroup sizes vary, these control limits and zone boundaries are only valid for the first observation, where n = 465. Each subgroup will have its own control limits and zone boundaries. Table 7.4 shows the results of calculating these in the same manner as for the first point.

Table 7.4
Control Limits and Zone Boundaries for Vehicles with Exact Change

Subgroup Number	n	Fraction Defective	UCL	LCL	Upper Zone C	Lower Zone C	Upper Zone B	Lower Zone B
1	465	0.387	0.468	0.332	0.423	0.377	0.446	0.354
2	123	0.309	0.533	0.267	0.444	0.356	0.488	0.312
3	309	0.460	0.484	0.316	0.428	0.372	0.456	0.344
4	83	0.241	0.561	0.239	0.454	0.345	0.508	0.292
5	116	0.302	0.536	0.264	0.445	0.355	0.491	0.309
6	306	0.353	0.484	0.316	0.428	0.372	0.456	0.344
7	333	0.571	0.481	0.319	0.427	0.373	0.454	0.346
8	265	0.400	0.490	0.310	0.430	0.370	0.460	0.340
9	354	0.266	0.478	0.322	0.426	0.374	0.452	0.348
10	256	0.453	0.492	0.308	0.431	0.369	0.461	0.339
11	406	0.458	0.473	0.327	0.424	0.376	0.449	0.351
12	415	0.359	0.472	0.328	0.424	0.376	0.448	0.352
13	379	0.237	0.475	0.325	0.425	0.375	0.450	0.350
14	341	0.434	0.480	0.320	0.427	0.373	0.453	0.347
15	258	0.415	0.491	0.309	0.430	0.370	0.461	0.339
16	270	0.311	0.489	0.311	0.430	0.370	0.46	0.340
17	480	0.385	0.467	0.333	0.422	0.378	0.445	0.355
18	350	0.526	0.479	0.321	0.426	0.374	0.452	0.348
19	433	0.485	0.471	0.329	0.424	0.376	0.447	0.353
20	479	0.411	0.467	0.333	0.422	0.378	0.445	0.355

We use these values to draw the control limits and zone boundaries in Figure 7.6. The process indicates many instances of a lack of control. Fully 25 percent of the subgroup proportions are out of control, and the data seem to be behaving in an extremely erratic pattern. Days 19, 18, 13, 9, and 7 are all beyond the control limits. Day 5 also indicates a lack of control because it is the second of three consecutive points falling in zone C or beyond on the same side of the centerline.

Figure 7.6
p Chart for Vehicles with Exact Change



Careful study is warranted to determine the cause or causes of this special variation. Removing the special causes of variation may require some fundamental changes in the way this system operates. Nevertheless, we must eliminate all of the special sources of variation before attempting to reduce the common causes of variation in the process.

Changing the Process. Management decides it would be advantageous to remove the erratic patterns in the preceding process. They could better serve the public by having adequate toll lanes of either the automatic or manned type available during rush hours. As a result of brainstorming, management institutes a third option for motorists; the use of electronic toll collection. Motorists using these are rewarded with a discount to encourage their use. Because the process has now been changed, a new set of observations is made. After a period of two months, to allow for transient effects to die down, the same sample selection method is again employed. Results for those subgroups appear in Table 7.5.

Table 7.5

Number of Vehicles Using Exact Change, Transponders or Tokens

Day	n	Number with Exact Change, Transponder or Token	Day	n	Number with Exact Change, Transponder or Token
1	421	171	11	401	199
2	466	197	12	384	165
3	389	192	13	428	213
4	254	107	14	352	149
5	186	89	15	444	193
6	456	189	16	357	158
7	411	211	17	283	147
8	322	139	18	424	207
9	287	136	19	337	143
10	262	131	20	326	157
			Totals	**7,190**	**3,293**

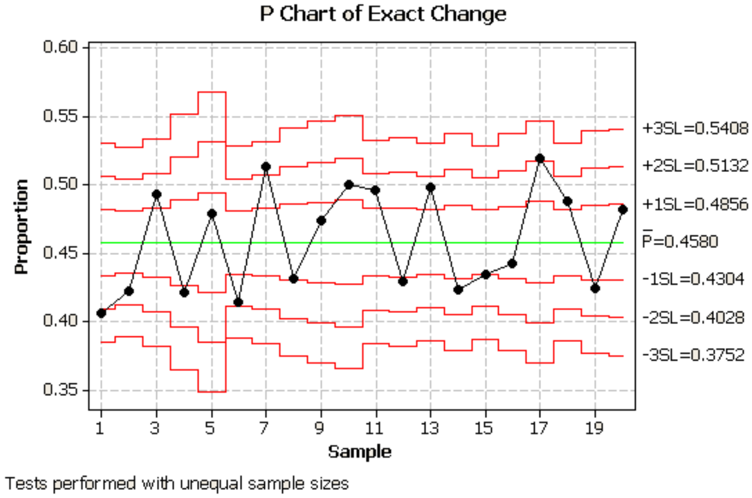
Their corresponding control limits and zone boundaries are shown in Table 7.6.

Table 7.6
Control Limits and Zone Boundaries for Vehicles with Exact Change

Subgroup Number	n	Fraction Defective	UCL	LCL	Upper Zone C	Lower Zone C	Upper Zone B	Lower Zone B
1	421	0.406	0.531	0.385	0.482	0.434	0.507	0.409
2	66	0.423	0.527	0.389	0.481	0.435	0.504	0.412
3	389	0.494	0.534	0.382	0.483	0.433	0.509	0.407
4	254	0.421	0.552	0.364	0.489	0.427	0.521	0.395
5	186	0.479	0.568	0.348	0.495	0.421	0.531	0.385
6	456	0.415	0.528	0.388	0.481	0.435	0.505	0.411
7	411	0.513	0.532	0.384	0.483	0.433	0.507	0.409
8	322	0.432	0.541	0.375	0.486	0.430	0.514	0.402
9	287	0.474	0.546	0.370	0.487	0.429	0.517	0.399
10	262	0.500	0.550	0.366	0.489	0.427	0.520	0.396
11	401	0.496	0.533	0.383	0.483	0.433	0.508	0.408
12	384	0.430	0.534	0.382	0.493	0.433	0.509	0.407
13	428	0.498	0.530	0.386	0.482	0.434	0.506	0.410
14	352	0.423	0.538	0.378	0.485	0.431	0.511	0.405
15	444	0.435	0.529	0.387	0.482	0.434	0.505	0.411
16	357	0.443	0.537	0.379	0.484	0.432	0.511	0.405
17	283	0.519	0.547	0.369	0.488	0.428	0.517	0.399
18	424	0.488	0.531	0.385	0.482	0.434	0.506	0.410
19	337	0.24	0.539	0.377	0.485	0.431	0.512	0.404
20	326	0.482	0.541	0.375	0.486	0.430	0.513	0.403

Figure 7.7 illustrates the control chart for the revised process. The process now appears stable, with no indications of a lack of control. Furthermore, the process is now improved: not only is the proportion of motorists using the exact change or transponder lanes stable and predictable, but the proportion of those motorists has risen from .400 to .458, which results in a smoother flow of traffic at the toll booths.

Figure 7.7
Revised Control Chart for Vehicles with Exact Change or Transponder



7.6 The np Chart

Classification data can sometimes be more easily understood if the data appear as counts rather than fractions. This is especially true when using attribute control charts to introduce control charting and encountering reluctance by some members of the affected community to deal with fractions rather than whole numbers, such as the number of defects.

The quantity np is the number of units in the subgroup with some particular characteristic, such as the number of nonconforming units. Traditionally, np charts are used only when subgroup sizes are constant. As the information used is the same as for p charts with constant subgroup sizes, these two charts are interchangeable.

Just as for the p chart, the categorization of data into two classes suggests, for a stable process, that every item must have approximately the same probability of being in one of the two categories. In a series of subgroups of constant size n, the mean or expected number of nonconforming items is approximated by np, and the associated standard error is given by $\sqrt{n\bar{p}\left(1-\bar{p}\right)}$. This enables us to construct the np chart.

7.6.1 Constructing the np Chart

Data collected for an np chart will be a series of integers, each representing the number of nonconforming (or conforming) items in its subgroup. Computations for the centerline, the control limits, and the zone boundaries are similar to those of the p chart with constant sample sizes.

The centerline is the overall average number of nonconforming (or conforming) items found in each subgroup of the data. For the ceramic tile importer discussed earlier in this chapter (the data appear in Table 7.1), there are a total of 183 cracked or broken tiles in the 30 subgroups examined; this represents an average count of 183/30 = 6.1 tiles per day; equivalently,

$$\text{Centerline (np)} = n\bar{p} = (100)\left[\frac{183}{3000}\right] = 6.100 \qquad (7.8)$$

The standard error is

$$\sqrt{n\bar{p}(1-\bar{p})} = \sqrt{(100)(0.061)(1-0.061)} = 2.393$$

Adding or subtracting three times the standard error from the centerline, respectively, yields the upper and lower control limits:

$$UCL(np) = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \qquad (7.9)$$

$$LCL(np) = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \qquad (7.10)$$

For the tile importer this yields values of

$$UCL(np) = (100)(0.061) + 3\sqrt{(100)(0.061)(1-0.061)} = 13.280$$

$$LCL(np) = (100)(0.061) - 3\sqrt{(100)(0.061)(1-0.061)} = -1.080$$

As the LCL value is negative (-1.080), and a negative value is meaningless, a value of 0 is used instead.

As for the p chart, the upper and lower boundaries between zones B and C are found by adding and subtracting one standard error from the centerline, $n\bar{p}$:

Boundary between upper zones B and C = $n\bar{p} + \sqrt{n\bar{p}(1-\bar{p})}$ (7.11)

Boundary between lower zones B and C = $n\bar{p} - \sqrt{n\bar{p}(1-\bar{p})}$ (7.12)

The upper boundary between zones B and C for this example is given by

$$6.1 + \sqrt{(100)(0.061)(1-0.061)} = 8.493$$

and the lower boundary between zones B and C is given by

$$6.1 - \sqrt{(100)(0.061)(1-0.061)} = 3.707$$

Upper and lower boundaries between zones B and A are found by adding and subtracting two standard errors from the centerline, $n\bar{p}$:

Boundary between upper zones A and B = $n\bar{p} + 2\sqrt{n\bar{p}(1-\bar{p})}$ (7.13)

Boundary between lower zones A and B = $n\bar{p} - 2\sqrt{n\bar{p}(1-\bar{p})}$ (7.14)

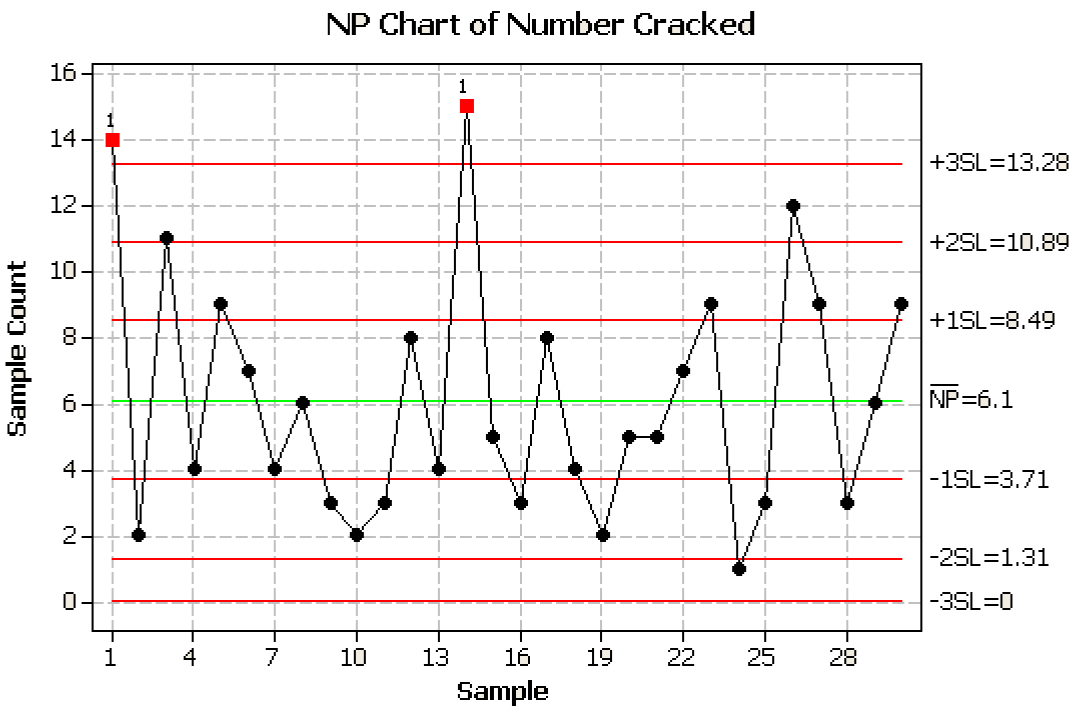
The results for this example are

$$6.1 + 2\sqrt{(100)(0.061)(1-0.061)} = 10.887$$

$$6.1 - 2\sqrt{(100)(0.061)(1-0.061)} = 1.313$$

Figure 7.8 illustrates the np chart for this process. A comparison of Figures 7.3 and 7.8 reveals that these two control charts are mathematically equivalent and present the same information. The only reason that one is preferred to the other is the form in which the data are presented, or the way in which the user prefers to visualize the control chart. Subsequent actions to stabilize the process are identical to those for the p chart.

Figure 7.8
np Chart for Cracked Tiles



7.7 Count Charts

A defective item is a nonconforming unit. It must be discarded, reworked, returned, sold, scrapped, or downgraded. It is unusable for its intended purpose in its present form. A defect, on the other hand, is an imperfection of some type that is undesirable, although it does not necessarily render the entire good or service unusable. One or more defects may not make an entire good or service defective. For example, we would not scrap or discard a computer, a washing machine, or an air conditioner because of a small scratch in the paint.

A car, air conditioner, bank form, or medical protocol may have one or more errors/defects that may not render the entire unit defective. However, the defect(s) may cause the unit to be downgraded, or necessitate its being reworked. In fact, there are many instances where more than one defect is the norm rather than the exception; this has created situations where products or services may not even be downgraded as a result of having several flaws.

Naturally, in the quest for improvement, our goal is no defects in our output. Control charting is one of the tools to help achieve this end.

When there are multiple opportunities for defects or imperfections in a given unit (such as a geographic area or a time period), we call each such unit an area of opportunity; each area of opportunity is a subgroup. When areas of opportunity are discrete units and a single defect will make the entire unit defective, a p chart or np chart is appropriate. But when areas of opportunity are continuous or very nearly so, and more than one defect may occur in a given area of opportunity, then a c chart or u chart should be used. The c chart is used when the areas of opportunity are of constant size, while the u chart is used when the areas of opportunity are not of constant size.

7.7.1 Conditions for Use

Area of opportunity control charts have wide applicability. If we are counting defects, the enamel on an appliance represents a continuous area of opportunity; a roll of cloth or plastic film is a continuous area of opportunity. If we are measuring the number of accidents recorded per week, a week represents a continuous area of opportunity. Measurements of the number of errors per hour in data entry or the number of typographical errors made per page have areas of opportunity (an hour or a page) that present enough opportunities for multiple defects to be considered nearly continuous. Imperfections in a complex piece of machinery, such as a computer, have areas of opportunity that are not strictly continuous; but the large number of individual components involved make the areas of opportunity close enough to continuous to be considered continuous.

If we are to use the c charts or u charts, the events we are studying must be describable as discrete events; these events must occur randomly within some well-defined area of opportunity; they should be relatively rare; and they should be independent of each other. Exact conformance to these conditions is not always easy to verify. Usually, it is not too difficult to tell whether the events are discrete and whether there is some well-defined area of opportunity. But whether the events are relatively rare is somewhat subjective and requires process knowledge and experience. The issue of independence is generally revealed by the control chart. That is, if the events are not random and independent, they will tend to form the identifiable special cause of variation patterns that we introduced in Chapter 6 and will discuss further in Chapter 9.

7.8 c Charts

The number of events in a constant area of opportunity is denoted by c; the count for each area of opportunity. The sequence of successive c values, taken over subgroups, is used to construct the c chart.

The centerline for the chart is the average number of events observed. It is calculated as

$$\text{Centerline(c)} = \bar{c} = \frac{\text{Total number of events observed}}{\text{Number of areas of opportunity}} \tag{7.15}$$

The standard error is the square root of the mean, $\sqrt{\bar{c}}$. Adding and subtracting three times the standard error from the centerline, $\bar{c}$, yields the upper and lower control limits. Thus

$$\text{UCL(c)} = \bar{c} + 3\sqrt{\bar{c}} \tag{7.16}$$

$$\text{LCL(c)} = \bar{c} - 3\sqrt{\bar{c}} \tag{7.17}$$

7.8.1 Counts, Control Limits, and Zones

As we have already seen with p charts and np charts, when a process is in a state of control, only very rarely will points fall beyond the control limits. Therefore, when a point does fall outside the control limits, we will consider it an indication of a lack of control and take appropriate action. When the lower control limit is calculated to be negative, we will use 0 as the lower control limit because, just as with p charts and np charts, negative numbers of events (such as -3 defects on a radio) are meaningless.

Consider a firm that has decided to use a c chart to help keep track of the number of telephone requests received daily for information on a given product. Each day represents an area of opportunity. Over a 30-day period, 1,206 requests are received, or an average of 40.2 per day; that is, $\bar{c}$ = 40.2. The upper and lower control limits can be found using Equations (7.16) and (7.17):

$$\text{UCL(c)} = 40.2 + 3\sqrt{40.2} = 59.2$$

$$\text{LCL(c)} = 40.2 - 3\sqrt{40.2} = 21.2$$

Actual counts occurring in an area of opportunity will always be whole numbers. Thus a count of 59 is within the control limits, while a count of 60 is beyond the UCL. The A, B, and C zone boundaries are constructed at one and two standard errors from the centerline, respectively. The zone boundaries are:

Boundary between lower zones B and C = $40.2 - \sqrt{40.2} = 33.9$

Boundary between lower zones A and B = $40.2 - 2\sqrt{40.2} = 27.5$

Boundary between upper zones A and B = $40.2 + 2\sqrt{40.2} = 52.9$

Boundary between upper zones B and C = $40.2 + \sqrt{40.2} = 46.5$

Because the actual counts are whole numbers, the observations would fall into zones as follows:

Zone	Counts
Upper A	53, 54, 55, 56, 57, 58, 59
Upper B	47, 48, 49, 50, 51, 52
Upper C	41, 42, 43, 44, 45, 46
Lower C	34, 35, 36, 37, 38, 39, 40
Lower B	28, 29, 30, 31, 32, 33
Lower A	22, 23, 24, 25, 26, 27

The zones each contain a reasonable number of whole numbers and are close enough in size to be workable. But consider the problem that would have been encountered if the process average had been $\bar{c}$ = 2.4. Here we would get

$$\mathrm{UCL(c)} = 2.4 + 3\sqrt{2.4} = 7.0$$

$$\mathrm{LCL(c)} = 2.4 - 3\sqrt{2.4} = -2.2 \ \text{(use 0.0)}$$

Boundary between lower zones B and C = $2.4 - \sqrt{2.4} = 0.9$

Boundary between lower zones A and B = $2.4 - 2\sqrt{2.4} = -0.7$ (use 0.0)

Boundary between upper zones A and B = $2.4 + 2\sqrt{2.4} = 5.5$

Boundary between upper zones B and C = $2.4 + \sqrt{2.4} = 3.9$

As before, because the counts are whole numbers, the observations will fall into zones as follows:

Zone	Counts
Upper A	6, 7
Upper B	4, 5
Upper C	3
Lower C	1, 2
Lower B	0
Lower A	0

These zones are so small that they are practically meaningless. The upper zone C, for example, only has one possible count, 3. When the average count is small, we generally do not make use of the zones in seeking indications of a lack of control. Rather, we focus on points beyond the control limits, runs of points above or below the centerline, and runs upward or downward in the data as indicators of a lack of stability. The exact value of the centerline, below which the use of A, B, and C zones becomes impractical, requires knowledge of, and experience with, the particular process involved. As a rule of thumb, the zone boundaries should not be used for c charts with average counts of less than 20.0.

Once again, there is no substitute for process knowledge and experience; however, as the observable count shrinks, the use of variables control charts must be instituted for continued process improvement.

Furthermore, keep in mind that when the average count is small, larger and larger areas of opportunity will be needed to detect imperfections. This will occur as a natural consequence of improved quality through the use of control charts. When the areas of opportunity needed to find imperfections grow unacceptably large, attribute control charts must be abandoned in favor of variables control charts which are discussed in Chapter 8. This usually solves the problem of small values for c and is another step on the ladder of quality consciousness.

7.8.2 Construction of a c Chart: An Example

Consider the output of a paper mill: the product appears at the end of a web and is rolled onto a spool called a reel. Every reel is examined for blemishes, which are imperfections. Each reel is an area of opportunity. Results of these inspections produce the data in Table 7.7.

Table 7.7

Number of Blemishes Found in 25 Reels of Paper

Reel	Number of Blemishes	Reel	Number of Blemishes
1	4	14	9
2	5	15	1
3	5	16	1
4	10	17	6
5	6	18	10
6	4	19	3
7	5	20	7
8	6	21	4
9	3	22	8
10	6	23	7
11	6	24	9
12	7	25	7
13	11	**Total**	**150**

The assumptions necessary for using the c chart are well met here, as the reels are large enough to be considered continuous areas of opportunity; imperfections are discrete events and seem to be independent of one another, and they are relatively rare. Even if these conditions are not precisely met, the c chart is fairly robust, or insensitive to small departures from the assumptions, so we may safely use it.

In this example the average number of imperfections per reel is

$$\text{Centerline(c)} = \bar{c} = \frac{150}{25} = 6.00$$

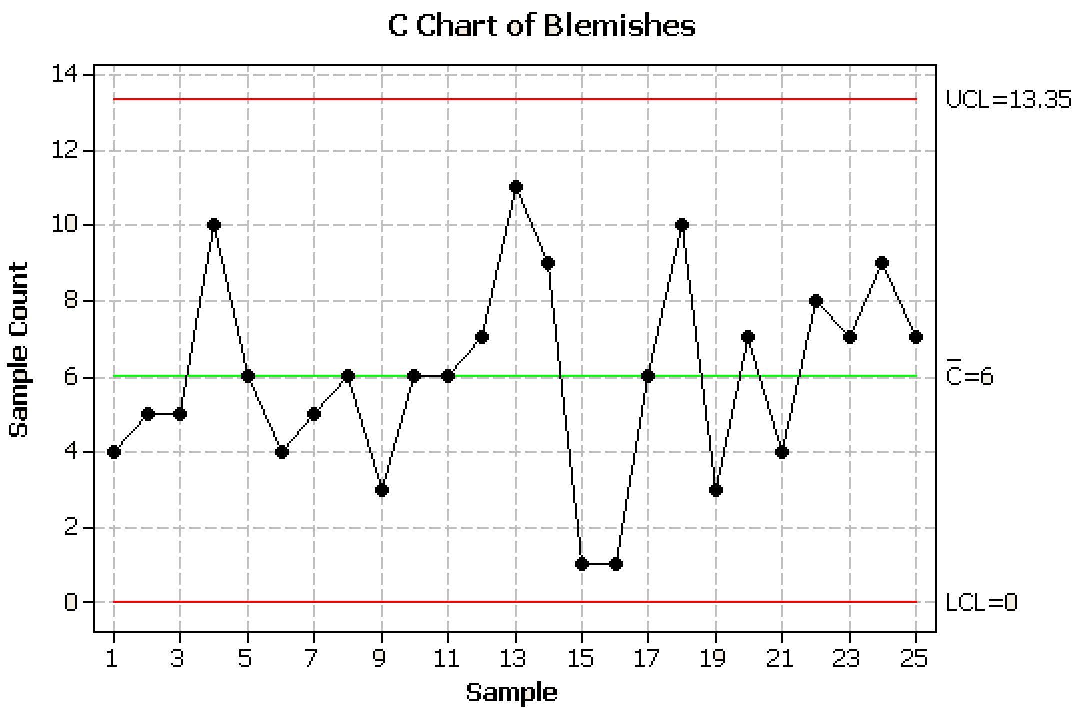
and the standard error is $\sqrt{6.00} = 2.45$.

Equations (7.16) and (7.17) yield upper and lower control limits:

$$\text{UCL}(c) = 6.00 + 3(2.45) = 13.35$$
$$\text{LCL}(c) = 6.00 - 3(2.45) = -1.35 \qquad (\text{use } 0.00)$$

The control chart is shown in Figure 7.9.

Figure 7.9
c Chart for Blemishes



The process is stable, with an average of 6 blemishes per reel, and the number of blemishes per reel is not likely to be 14 or more.

7.8.3 Small Average Counts

Even though the control chart in Figure 7.9 is useful, frequently, when average counts are small, data appearing as counts will tend to be asymmetric. This may lead to over-adjustment (false alarms) or under-adjustment (too little sensitivity).

False alarms are indications that the process is exhibiting special variation when no special variation can be found. Most often, these indications will be points on the control chart that are just beyond the upper control limit. False alarms, in and of themselves, can destabilize a stable process. Employees searching for special sources of variation will generally fix something that does not need fixing. That is, they will adjust the process to compensate for nonexistent special sources of variation. This may send the system into a complete state of chaos. Also, false alarms may demoralize employees who may begin to feel that many of their efforts do not result in process improvements.

In some cases, control limits calculated using Equations (7.16) and (7.17) may not provide sufficient sensitivity to an indication of a special source of variation. This can result in a loss of opportunity for process improvement.

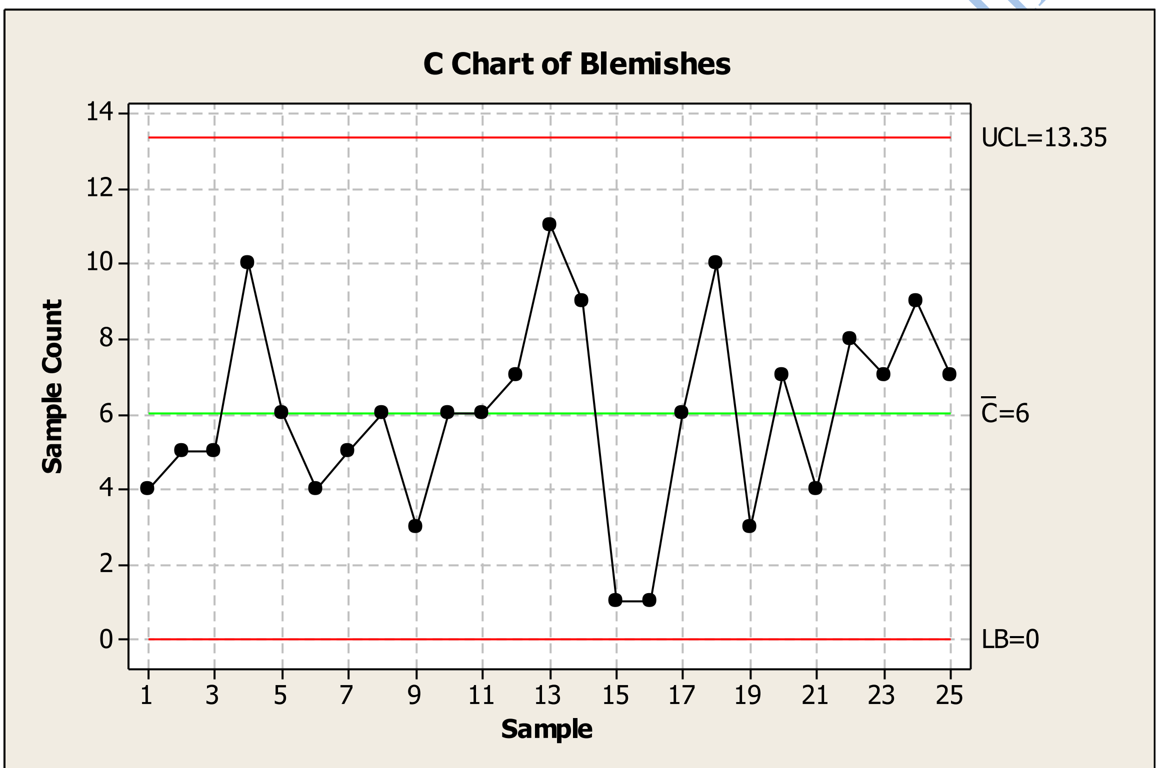
To avoid both of these problems, we may use a set of **fixed control limits** for the c chart. These fixed control limits are sometimes called **probability control limits** and provide an excellent and economical rule for separating special and common variation when average counts are less than 20. Table 7.8 gives values for upper and lower control limits that can be used when average counts are less than 20.

Table 7.8
c chart Using Fixed Control Limits

Process Average	LCL	UCL	Process Average	LCL	UCL
0 to 0.10	0	1.5	9.65 to 10.35	2.5	19.5
0.11 to 0.33	0	2.5	10.36 to 10.97	2.5	20.5
0.34 to 0.67	0	3.5	10.98 to 11.06	3.5	20.5
0.68 to 1.07	0	4.5	11.07 to 11.79	3.5	21.5
1.08 to 1.53	0	5.5	11.80 to 12.52	3.5	22.5
1.54 to 2.03	0	6.5	12.53 to 12.59	3.5	23.5
2.04 to 2.57	0	7.5	12.60 to 13.25	4.5	23.5
2.58 to 3.13	0	8.5	13.26 to 13.99	4.5	24.5
3.14 to 3.71	0	9.5	14.00 to 14.14	4.5	25.5
3.72 to 4.32	0	10.5	14.15 to 14.74	5.5	25.5
4.33 to 4.94	0	11.5	14.75 to 15.49	5.5	26.5
4.95 to 5.29	0	12.5	15.50 to 15.65	5.5	27.5
5.30 to 5.58	0.5	12.5	15.66 to 16.24	6.5	27.5
5.59 to 6.23	0.5	13.5	16.25 to 17.00	6.5	28.5
6.24 to 6.89	0.5	14.5	17.01 to 17.13	6.5	29.5
6.90 to 7.43	0.5	15.5	17.14 to 17.76	7.5	29.5
7.44 to 7.56	1.5	15.5	17.77 to 18.53	7.5	30.5
7.57 to 8.25	1.5	16.5	18.54 to 18.57	7.5	31.5
8.26 to 8.94	1.5	17.5	18.58 to 19.36	8.5	31.5
8.95 to 9.27	1.5	18.5	19.37 to 20.00	8.5	32.5
9.28 to 9.64	2.5	18.5			

In the example of the reels of paper, the centerline is 6.00. Therefore, for this application of the c chart the control limits should properly have come from Table 7.8. As 6.00 is in the 5.59 to 6.23 range, the values for the lower and upper control limits respectively are 0.5 and 13.5. These values have been used to draw the c chart in Figure 7.10. Note that for this control chart the Minitab software has to have the option selected that permits the limits to be set manually. This can be found under the c chart options and then selecting the tab for “s limits”.

Figure 7.10
c Chart Using Fixed Probability Limits for Number of Blemishes



Notice that these control limits are almost the same as those created using Equations (7.16) and (7.17): 13.35 and 0.00. In general, because the number of events is a whole number, both control charts may show the same indications of a lack of control. In this particular case, the resulting control charts are similar, but a count of 0 will indicate a lack of control using the fixed limits, and will not indicate a lack of control using the three-sigma limits.

It is not too unusual to find that the control limits resulting from computations using Equations (7.16) and (7.17) and those resulting from Table 7.8 are similar, and some users merely ignore these table values. The danger in doing so, however, is that when average counts are small, the three-sigma limits may generate false indications of a lack of control or fail to signal a lack of control. This can lead to over-adjustment or under-adjustment of a process, which in and of itself may cause the process to become out of control or may lead to frustration on the part of those employees trying to search for special causes of variation where none exist.

A note of caution when dealing with c charts: those charged with determining the number of imperfections must be clear and consistent in the definition of an imperfection. Operational definitions, as discussed in Chapter 4, are extremely important, and the individuals identifying imperfections must be properly trained so that they understand the nature of the process; if they are not, some identified imperfections may not actually be imperfections, while some actual imperfections may go undetected -- hence the independence of observations of the occurrence of imperfections may suffer. This, in turn, may result in violations of the underlying assumptions used for the c chart, resulting in either the generation of many false alarms, or in undetected out-of-control behavior.

7.8.4 Stabilizing a Process: An Example

An industrial washing machine manufacturer inspects completed units for defects. Table 7.9 lists counts of defects found on 24 machines.

Table 7.9

Defects Found on 24 Machines

Machine Number	Count	Machine Number	Count
1	62	13	51
2	60	14	75
3	36	15	49
4	39	16	52
5	36	17	62
6	47	18	43
7	33	19	70
8	32	20	18
9	74	21	44
10	71	22	20
11	43	23	18
12	39	24	26
		Total	**1,100**

The centerline for the control chart is

$$\bar{c} = 1100 / 24 = 45.8$$

and the control limits can be found using Equations (7.16) and (7.17):

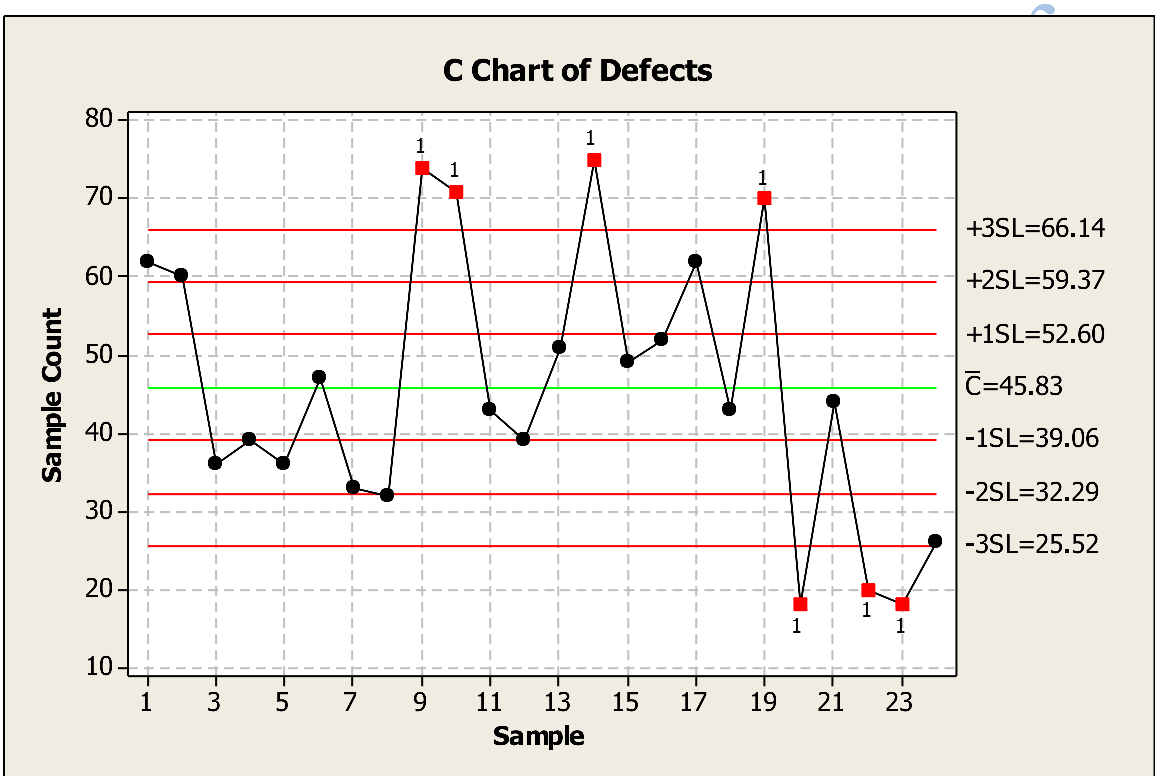
$$LCL(c)=45.8-3\sqrt{45.8}=25.5$$

and

$$UCL(c)=45.8+3\sqrt{45.8}=66.1$$

The completed control chart, including zones, is shown in Figure 7.11. Counts of 67 or more and 25 or fewer indicate a lack of control.

Figure 7.11
c Chart for Defects in Industrial Washing Machines



The process is not in control. Special causes of variation are present. Let us assume that the local operators responsible for the final inspection act so that the special causes of variation for points 9, 10, 14, 19, 20, 22, and 23 are identified and the appropriate corrections are made. The data for points affected by known special causes that have been eliminated are deleted from the data set, and the centerline and control limits are recomputed:

$$\bar{c}=\frac{754}{17}=44.4$$

$$LCL(c)=44.4-3\sqrt{44.4}=24.4$$

$$UCL(c)=44.4+3\sqrt{44.4}=64.4$$

The new limits are so close to the old limits that the old limits are used for the next 24 machines produced; Table 7.10 presents the data.

Table 7.10
Defects Found on Next 24 Industrial Washing Machines

Machine Number	Count	Machine Number	Count
25	21	37	46
26	18	38	31
27	7	39	42
28	12	40	44
29	18	41	26
30	32	42	37
31	32	43	26
32	37	44	29
33	39	45	31
34	39	46	34
35	34	47	36
36	39	48	40

The first five data points for these next 24 machines are well below the lower control limit. Investigation by the local operators reveals that a substitute for the regular inspector counted the defects on those five machines. The substitute was not properly trained and did not identify all the defects correctly. The operators informed management, and management made appropriate changes in policy so that this situation would not recur. These points can now be eliminated from the data. Beginning with machine number 30, all counts are below the process average. Local operators decided that the process has been changed, so a revised control chart is constructed beginning with point number 30, as shown in the data file WASHING2.

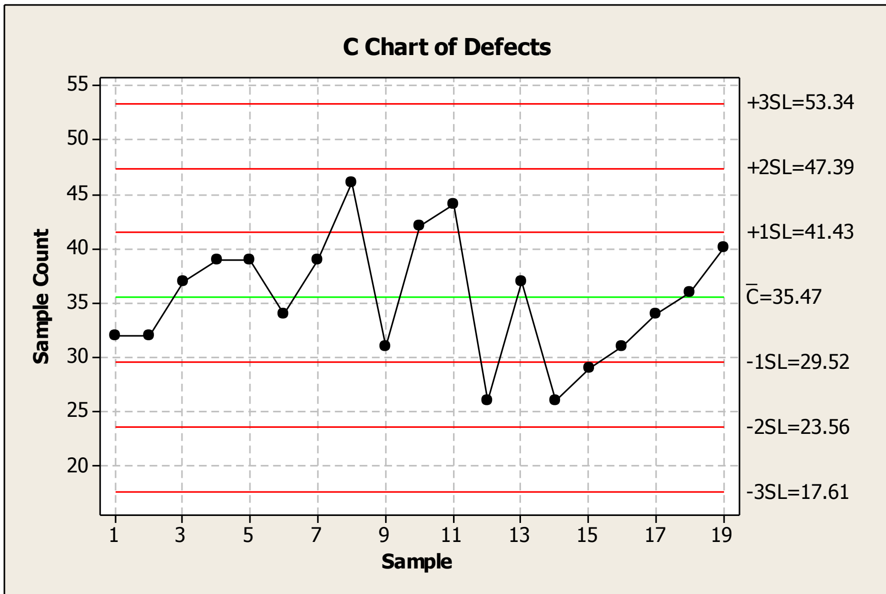
$$\bar{c} = \frac{674}{19} = 35.5$$

$$\text{LCL}(c) = 35.5 - 3\sqrt{35.5} = 17.6$$

$$\text{UCL}(c) = 35.5 + 3\sqrt{35.5} = 53.4$$

Figure 7.12 illustrates the revised control chart. The process, as it stands, now appears to be in a state of control.

Figure 7.12
Revised c Chart for Defects in Industrial Washing Machines



7.8.5 Construction of a c chart: Another Example

Consider the case of a mill with a constant work force of 450 employees that has a sign posted at the employee entrance reading "SAFETY IS BETTER THAN COMPENSATION." Informal conversations with employees reveal they consider the sign a reminder to be careful. But management has not simultaneously made the work environment safer. There are still cluttered aisles, and spills and leaks of liquids on the floor are not attended to rapidly. The workers know this but have long ago stopped their fruitless efforts at getting management to allocate the resources necessary to create a safer workplace.

To examine the problem, a control chart of the number of accidents per month is constructed. Table 7.11 shows the data for the past 26 months.

Table 7.11

Accidents per Month

Month	Number of Accidents	Month	Number of Accidents
Jan	3	Feb	2
Feb	2	Mar	0
Mar	0	Apr	0
Apr	2	May	3
May	1	Jun	2
Jun	1	Jul	0
Jul	1	Aug	1
Aug	0	Sep	0
Sep	0	Oct	1
Oct	1	Nov	0
Nov	1	Dec	0
Dec	3	Jan	1
Jan	0	Feb	1
		Total	**26**

Note that as the number of labor hours per month remains constant, the area of opportunity is considered constant month-to-month. The centerline for the c chart is:

$$\bar{c} = \frac{26}{26} = 1.00$$

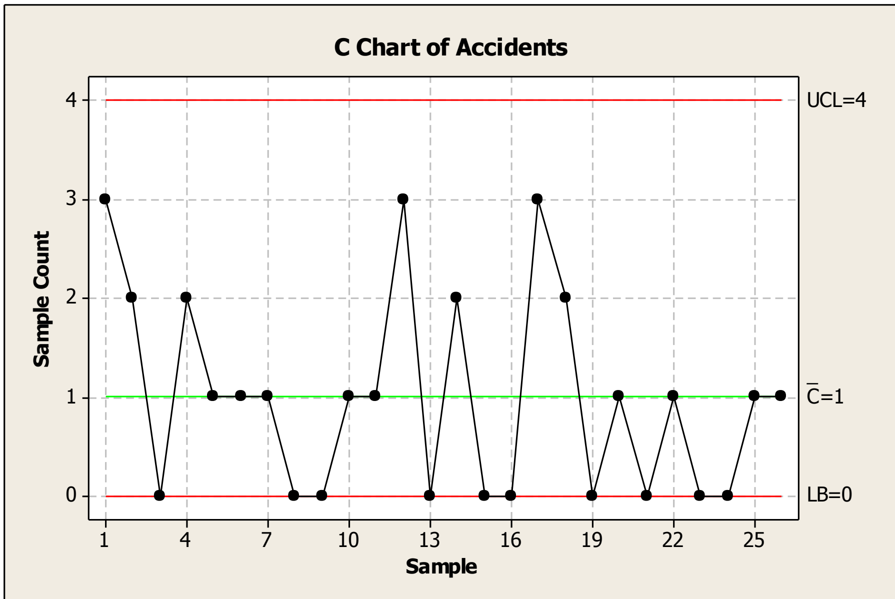
From Table 7.8, the fixed control limits are:

LCL(c) = 0 and UCL(c) = 4.5

Figure 7.13 displays the control chart constructed using Minitab, consequently, the UCL = 4.0, not 4.5 as is required when using fixed probability limits due to an average defect count of 2.0 or less. In any event, there are no indications of any special variation so we conclude that the process is in a state of statistical control.

The company may not realize it, but it is in the business of producing accidents at the stable rate of one per month. It will continue to do so until some effort is made to change the underlying process. If no change in the process is made, accidents will continue to be produced at this rate. Consequently, the "SAFETY IS BETTER THAN COMPENSATION" sign is unfair: employees are not empowered to make system changes that would lower the average number of accidents per month; the sign subtly and unjustly shifts the burden of responsibility for safety from management to the employees.

Figure 7.13
Control Chart for Accidents per Month



7.8.6 Construction of a c Chart: Another Example

A national company opens a sales office with six salesmen in Cleveland, Ohio. [Gitlow and Gitlow, 1987, pp. 124-125] The office has been open for just over six months. All salesmen have the same responsibilities and opportunities. The number of new accounts generated by each salesman in the first six months of operation is recorded in Table 7.12.

Table 7.12
Number of New Accounts

Name of Salesperson	Number of New Accounts
Allan	27
Fred	36
Mark	28
David	24
John	29
Phil	30
Total	**174**

The company's policy calls for a semi-annual review of performance to determine who should be rewarded or punished. Based on traditional thinking, it appears that the company should reward Fred and punish David.

Examining the above data from the perspective of the System of Profound Knowledge and statistical thinking results in the following c chart statistics:

$$\bar{c} = 174/6 = 29$$

$$UCL = 29 + 3[\sqrt{29}] = 45.2 \rightarrow 45$$

$$LCL = 29 - 3[\sqrt{29}] = 12.8 \rightarrow 13$$

According to the above calculations, the number of new accounts generated by a salesman in a six-month period can be predicted to be between 13 and 45 new accounts. This large amount of variation is attributable to random noise (common causes of variation) in the sales system. All salesmen are in the same sales system, and they all deserve the percentage pay raise. No one should be rewarded, and no one should be punished. Sales management should focus their attention on improvement of the sales system, not on rewarding and punishing salesmen.

If Fred had generated 66 new accounts (instead of 36) the sales system's statistics would be:

$$\bar{c} = 204/6 = 34$$

$$UCL = 34 + 3[\sqrt{34}] = 51.5 \rightarrow 51$$

$$LCL = 34 - 3[\sqrt{34}] = 16.5 \rightarrow 17$$

In this scenario, Fred is outside the sales system on the high side (he is above 51). Investigation by Fred's manager leads to the realization that Fred has developed a better telephone procedure for screening potential customers. Fred should receive special recognition because he is outside the sales system on the high side and because his efforts provide guidance for improvement for all salesmen all in the sales system.

7.9 u Charts

In some applications of count charts, the areas of opportunity vary in size. Generally, the construction and interpretation of control charts are easier when the area of opportunity remains constant, but from time to time variation in the subgroup size, or area of opportunity, may be unavoidable. For example, samples taken from a roll of paper may need to be manually torn from rolls, so that the areas sampled -- the areas of opportunity -- will vary; continuous welds in heat exchangers will have varying areas of opportunity depending on the total number and lengths of the welds present in different units; and the number of word processing errors in a document will have areas of opportunity that will vary with the lengths of the documents. When the areas vary, the appropriate control chart to be used is a u chart.

The u chart is similar to the c chart in that it is a control chart for the count of the number of events, such as the number of nonconformities over a given area of opportunity. The fundamental difference lies in the fact that during construction of a c chart, the area of opportunity remains constant from observation to observation, while this is not a requirement for the u chart. Instead, the u chart considers the number of events (such as blemishes or other defects) as a fraction of the total size of the area of opportunity in which these events were possible, thus circumventing the problem of having different areas of opportunity for different observations.

The characteristic used for the control chart, u, is the ratio of the number of events to the area of opportunity in which the events may occur. For observation i, we call the number of events (such as imperfections) the observed c_i, and the area of opportunity a_i. Thus, u_i is the ratio

$$u_i = \frac{c_i}{a_i} \qquad (7.18)$$

for each point.

The average of all the u_i values, $\bar{u}$, provides a centerline for the control chart:

$$\text{Centerline } (u) = \bar{u} = \frac{\sum c_i}{\sum a_i} \qquad (7.19)$$

Control limits are usually placed at three standard errors on either side of the centerline for each individual subgroup. The standard error is given by the square root of the average u value divided by the subgroup's area of opportunity, $\sqrt{\bar{u}/a_i}$.

Since the area of opportunity varies from subgroup to subgroup, so does the standard error. This results in control limits that vary from subgroup to subgroup:

$$\text{LCL}(u) = \bar{u} - 3\sqrt{\frac{\bar{u}}{a_i}} \qquad (7.20)$$

$$\text{UCL}(u) = \bar{u} + 3\sqrt{\frac{\bar{u}}{a_i}} \qquad (7.21)$$

When the lower control limit is negative, a value of 0.0 is used instead.

7.9.1 Construction of a u Chart: An Example

Consider the case of the manufacture of a certain grade of plastic. The plastic is produced in rolls, with samples taken five times daily. Because of the nature of the process, the square footage of each sample varies from inspection lot to inspection lot. Hence the u chart should be used here. Table 7.13 shows the data on the number of defects, c_i, for the past 30 inspection lots. The number of defects per 100 square feet is calculated from Equation (7.18).

Table 7.13

Defects in Rolls of Plastic

Inspection Lot (i)	Square Feet of Plastic	Area of Opportunity (in 100 square feet) a_i	Number of Defects in Lot c_i	Defects per 100 Square Feet u_i
1	200	2.00	5	2.50
2	250	2.50	7	2.80
3	100	1.00	3	3.00
4	90	0.90	2	2.22
5	120	1.20	4	3.33
6	80	0.80	1	1.25
7	200	2.00	10	5.00
8	220	2.20	5	2.27
9	140	1.40	4	2.86
10	80	0.80	2	2.50
11	170	1.70	1	0.59
12	90	0.90	2	2.22
13	200	2.00	5	2.50
14	250	2.50	12	4.80
15	230	2.30	4	1.74
16	180	1.80	4	2.22
17	80	0.80	1	1.25
18	100	1.00	2	2.00
19	140	1.40	3	2.14
20	120	1.20	4	3.33
21	250	2.50	2	0.80
22	130	1.30	3	2.31
23	220	2.20	1	0.45
24	200	2.00	5	2.50
25	100	1.00	2	2.00
26	160	1.60	4	2.50
27	250	2.50	12	4.80
28	80	0.80	1	1.25
29	150	1.50	5	3.33
30	210	2.10	4	1.90
Totals	**4,790**	$\Sigma a_i = 47.90$	$\Sigma c_i = 120$	

Using Equation (7.19), we find the centerline to be:

$$\text{Centerline(u)} = \text{Average number of defects/100 sq. ft.} = \bar{u} = 120/47.90 = 2.51$$

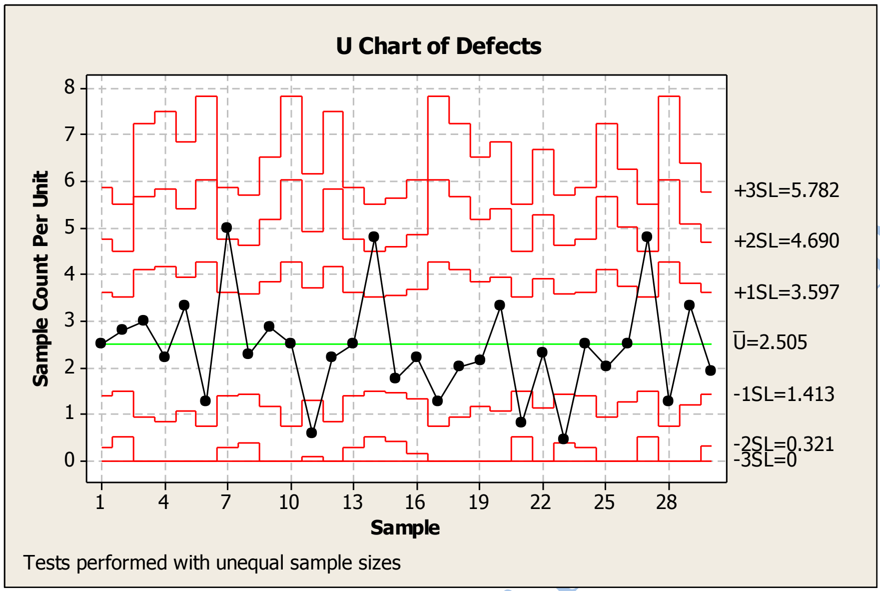
The control limits are different for each of the subgroups and must be computed individually for each subgroup using Equations (7.20) and (7.21). Table 7.14 shows the resulting values.

Table 7.14
Control Limits for Defects in Rolls of Plastic

Inspection Lot i	Number of Inspection Units a_i	LCL	UCL	Inspection Lot i	Number of Inspection Units a_i	LCL	UCL
1	2.0	0	5.9	16	1.8	0	6.1
2	2.5	0	5.5	17	0.8	0	7.8
3	1.0	0	7.3	18	1.0	0	7.3
4	0.9	0	7.5	19	1.4	0	6.5
5	1.2	0	6.8	20	1.2	0	6.8
6	0.8	0	7.8	21	2.5	0	5.5
7	2.0	0	5.9	22	1.3	0	6.7
8	2.2	0	5.7	23	2.2	0	5.7
9	1.4	0	6.5	24	2.0	0	5.9
10	0.8	0	7.8	25	1.0	0	7.3
11	1.7	0	6.2	26	1.6	0	6.3
12	0.9	0	7.5	27	2.5	0	5.5
13	2.0	0	5.9	28	0.8	0	7.8
14	2.5	0	5.5	29	1.5	0	6.4
15	2.3	0	5.6	30	2.1	0	5.8

For u charts, the manual calculation of zone boundaries is computationally cumbersome; however, Minitab easily incorporates these zones. Figure 7.14 shows the Minitab control chart, with zones, for this process. No points indicate a lack of control, so there is no reason to believe that any special variation is present. If sources of special variation were detected, we would proceed as we did with the c chart -- that is, we would identify the source or sources of the special variation, eliminate them from the system if detrimental, or incorporate them into the system if beneficial; drop the data points from the data set; and reconstruct and reanalyze the control chart.

Figure 7.14
u Chart for Number of Defectives in Rolls of Plastic



7.10 Limitations of Attribute Control Charts

As processes improve and defects or defectives become rarer, the number of units that must be examined to find one or more of these events increases. If we consider a p chart where the average fraction of nonconforming items is 0.005 (0.5%), then on average we would need to examine 200 units to have an average count of just 1.00. In the extreme, to maintain a reasonable average count as the area of opportunity grows, 100 percent inspection becomes the rule. This implies inspecting all of the items and sorting those that conform to some specification from those that do not. Not only is this inspection costly, but it is equivalent to accepting the fact that the process is producing a constant fraction of its output as defective and will continue to do so. Hence, attribute control charts are limited in terms of the level of process improvement they enable. Additional process improvement, however, is possible with variables control charts, to be discussed in Chapter 8.

Another disadvantage of using attribute control charts is that if special variation from several different sources is present, it is difficult to identify and isolate the special sources individually. One or more of these special sources may mask another, resulting in a process that appears to be stable but is really operating under the influence of several special sources of variation. As the number of special sources of variation increases, their tendency to mask one another can grow, resulting in further difficulties in the future. On the other hand, variables control charts use numerical measurements, which make them more revealing and powerful than attribute control charts. Attribute data indicate only whether a given unit conforms; they fail to reveal by how much a unit is beyond an upper or lower specification limit. Therefore, attribute data will not provide as clear a direction for process improvement as will variables data.

7.11 Summary

Attribute control charts can be broadly classified into two groups: (1) p charts and np charts based on percent or number of defectives, and (2) c charts and u charts based on counts over areas of opportunity.

The p chart can help stabilize a process by indicating a lack of statistical control in some characteristic of interest measured as a proportion of output, such as fraction defective. Subgroup sizes may be constant or may vary from subgroup to subgroup.

The np chart is mathematically equivalent to a p chart; however, the number, rather than the proportion of items, with the characteristic of interest is charted. Subgroup sizes are generally held constant for each subgroup for the np chart.

A c chart is used when a single unit of output may have multiple events, such as the number of defects in an appliance or in a roll of paper. The c chart helps stabilize the number of events when the area of opportunity in which the events may occur remains the same for each unit of output from the system.

The u chart is used when the area of opportunity varies from unit to unit, and counts of the number of events are to be control charted.

While attribute control charts help identify special process variation, as the process improves and the number of defectives or defects becomes smaller, the subgroup size necessary to detect these events becomes prohibitively large. Hence, the use of attribute charts is only a milestone on the road to never-ending improvement. To continue on the journey, variables control charts must be instituted and used.

EXERCISES

7.1 A manufacturer of wood screws periodically examines screw heads for the presence or absence of burrs. Subgroups of 300 screws are selected and examined using a carefully designed procedure. The data appear below.

BURR

Observation	Number of screws with burrs
1	19
2	16
3	11
4	6
5	22
6	2
7	4
8	7
9	5
10	27
11	7
12	13
13	17
14	29
15	1
16	2
17	19
18	28
19	24
20	23

a. Find the centerline and standard error.
b. Find the control limits and zone boundaries.
c. Are there any special causes of variation? Which observations?

7.2 A large metropolitan hospital processes many samples of blood daily. Some occasionally get mislabeled or lost, so new samples are required (rework). Subgroups of 50 samples are tracked each day for a 30-day period. Construct a control chart to search for special sources of variation.

BLOOD

Sample Number	n	Missing or Lost	Sample Number	n	Missing or Lost	Sample Number	n	Missing or Lost
1	50	4	11	50	4	21	50	3
2	50	4	12	50	2	22	50	1
3	50	6	13	50	3	23	50	0
4	50	5	14	50	0	24	50	4
5	50	2	15	50	2	25	50	3
6	50	0	16	50	3	26	50	5
7	50	6	17	50	5	27	50	6
8	50	1	18	50	1	28	50	2
9	50	2	19	50	6	29	50	3
10	50	0	20	50	5	30	50	1

7.3 A firm with 248 vehicles at one location keeps track of the number out of service each day. Out of service is defined as unavailable for normal use for four or more hours.

VEHICLE

Day	Number out of Service	Proportion	N	Day	Number out of Service	Proportion	n
1	7	0.028	248	14	6	0.024	248
2	3	0.012	248	15	2	0.008	248
3	6	0.024	248	16	5	0.020	248
4	2	0.008	248	17	8	0.032	248
5	1	0.004	248	18	9	0.036	248
6	0	0.000	248	19	3	0.012	248
7	12	0.048	248	20	1	0.004	248
8	3	0.012	248	21	0	0.000	248
9	6	.0.024	248	22	2	0.008	248
10	4	0.016	248	23	4	0.016	248
11	3	0.012	248	24	6	0.024	248
12	2	0.008	248	25	4	0.016	248
13	7	0.028	248				

a. Determine the centerline and control limits for the appropriate control chart.
b. Are there any indications of a lack of control? What are the indications, and why do they indicate a lack of control?

7.4 A bank is studying the proportion of transactions made using an ATM. The following represents a series of days and the number of transactions using the ATM.

ATM

Day	Total Number of Transactions	Number of ATM Transactions	Day	Total Number of Transactions	Number of ATM Transactions
1	320	87	14	312	91
2	356	92	15	390	131
3	280	75	16	354	78
4	325	109	17	322	89
5	344	69	18	353	98
6	410	99	19	317	81
7	385	120	20	374	58
8	324	86	21	409	104
9	367	111	22	366	81
10	312	90	23	298	87
11	276	86	24	31	72
12	342	106	25	339	84
13	387	65			

a. Determine the centerline and control limits for the appropriate control chart.
b. Are there any indications of a lack of control? What are the indications, and why do they indicate a lack of control?

7.5 A company manufactures 2,000 lawn mowers per day. Every day 40 lawn mowers are randomly selected from the production line. If a mower fails to start on the first pull, it's labeled "nonconforming." Results for 22 days are shown below.

MOWER

Day	n	Number Nonconforming	Day	n	Number Nonconforming
1	40	2	12	40	4
2	40	3	13	40	7
3	40	1	14	40	2
4	40	4	15	40	3
5	40	3	16	40	3
6	40	2	17	40	2
7	40	1	18	40	8
8	40	1	19	40	0
9	40	0	20	40	1
10	40	3	21	40	3
11	40	2	22	40	2

a. Find the centerline for the appropriate control chart.
b. Determine the control limits and zone boundaries.
c. Find any indications of a lack of control.

7.6 A given model of a large radar dish represents an area of opportunity in which non-conformities may occur. Results for 25 such assemblies are shown below.

RADAR

Assembly Number	Number of Nonconformities	Assembly Number	Number of Nonconformities	Assembly Number	Number of Nonconformities
1	25	10	44	19	21
2	60	11	62	20	88
3	28	12	81	21	34
4	65	13	70	22	82
5	91	14	75	23	53
6	56	15	24	24	102
7	40	16	50	25	64
8	54	17	70		
9	90	18	56		

a. Determine the centerline and control limits for the appropriate control chart.
b. Are there any indications of a lack of control? What are the indications, and why do they indicate a lack of control?

7.7 Samples of 90 retainer rings are examined for the number nonconforming. The results for 30 consecutive days are shown below.

COPPER

Day	Number Nonconforming	Day	Number Nonconforming	Day	Number Nonconforming
1	11	11	14	21	11
2	8	12	13	22	11
3	3	13	12	23	8
4	7	14	1	24	7
5	13	15	8	25	1
6	5	16	14	26	4
7	15	17	16	27	14
8	13	18	6	28	11
9	14	19	2	29	15
10	15	20	10	30	12

a. Determine the centerline and control limits.
b. Are there any indications of a lack of control? On which days?

7.8 A large publisher counts the number of keyboard errors that make their way into finished books. The number of errors and the number of pages in the past 26 publications are shown below.

BOOK

Book Number	Number of Errors	Number of Pages	Book Number	Number of Errors	Number of Pages
1	49	202	14	48	612
2	63	232	15	50	432
3	57	332	16	41	538
4	33	429	17	45	383
5	54	512	18	51	302
6	37	347	19	49	285
7	38	401	20	38	591
8	45	412	21	70	310
9	65	481	22	55	547
10	62	770	23	63	469
11	40	577	24	33	652
12	21	734	25	14	343
13	35	455	26	44	401

a. Determine the centerline and control limits for the appropriate control chart.
b. Are there any indications of a lack of control? For which books?

7.9 Lots of cloth produced by a manufacturer are inspected for defects. Because of the nature of the inspection process, the size of the inspection sample varies from lot to lot.

CLOTH

Lot Number	100's of Square Yards	Number of Defects	Lot Number	100's of Square Yards	Number of Defects
1	2.0	5	9	1.9	3
2	2.5	7	10	1.5	0
3	1.0	3	11	1.7	2
4	0.9	2	12	1.7	3
5	1.2	4	13	2.0	1
6	0.8	1	14	1.6	2
7	1.4	0	15	1.9	4
8	1.6	2			

a. Calculate the centerline and upper and lower control limits for the appropriate control chart.
b. Are any special causes of variation present in the data? For which lots?

7.10 A firm manufactures high voltage capacitor film for the electronics industry. They are concerned with the yield on a slitter process which produces reels of film. Reels that are nonconforming must be scraped. Reels were sampled and inspected each week for 38 weeks., with the number of reels of film in each sample, number of scrap reels and proportion of scrap reels shown below.

REELS

Proportion of

Week	Total Reels	Number of Scrap Reels	Number of Scrap Reels
1	1,145	142	0.1240
2	1,013	55	0.0543
3	1,275	125	0.0980
4	686	57	0.0831
5	984	58	0.0589
6	717	37	0.0516
7	1,408	57	0.0405
8	1,254	38	0.0303
9	890	60	0.0674
10	1,155	99	0.0857
11	969	121	0.1249
12	858	69	0.0804
13	832	100	0.1202
14	839	101	0.1204
15	1,230	123	0.1000
16	843	49	0.0581
17	1,102	99	0.0898
18	1,039	111	0.1068
19	1,385	125	0.0903
20	1,352	142	0.1050
21	903	43	0.0476
22	976	64	0.0656
23	695	81	0.1165
24	1,123	82	0.0730
25	1,252	102	0.0815
26	857	113	0.1319
27	1,277	74	0.0579
28	1,182	97	0.0821
29	440	41	0.0932
30	916	123	0.1343

a. Construct an appropriate control chart for this data.
b. Is the process in a state of statistical control? Why?

7.11 A computer systems team is concerned about a data management system. Each day decisions need to be made that relate to the way in which records and data files are to be maintained. Some information is required to be maintained for one day, some for seven days, some for thirty days, some for a year, and some for perpetual storage. Data that is to be maintained for more than seven days must be stored remotely on disk cartridges off the production site. The data cartridges used for the remote off-site storage involve both an acquisition expense and the cost of maintaining and managing the remote storage system. In an effort to study the stability of the process, weekly reports for the past six months are obtained. The number of data cartridges sent for remote storage each week during this period are presented in the table below.

CARTS

Week	Data Cartridges Sent
1	123
2	116
3	115
4	116
5	115
6	120
7	140
8	137
9	141
10	142
11	164
12	148
13	160
14	134
15	162
16	174
17	174
18	176
19	193
20	173
21	147
22	159
23	147
24	147

a. Construct an appropriate control chart for these data.
b. Is the process in a state of statistical control? Why?
c. If the process is not in a state of control, eliminate out of control points and recalculate the trial control limits.

7.12 A medical transcription service enters medical data on patient files for hospitals. The service studied ways to improve the turnaround time (defined as the time between receiving the data and the time the client receives the completed files). After studying the process, it is determined that turnaround time is increased by transmission errors. A transmission error is defined as data transmitted that does not go through as planned, and needs to be retransmitted. Each day a sample of 125 record transmissions is randomly selected and evaluated for errors. The table below presents the number and proportion of transmissions with errors in samples of 125 records transmitted.

TRANSMIT

Date	Number of Errors	Proportion of Errors
August:		
1	6	0.048
2	3	0.024
5	4	0.032
6	4	0.032
7	9	0.072
8	0	0.000
9	0	0.000
12	8	0.064
13	4	0.032
14	3	0.024
15	4	0.032
16	1	0.008
19	10	0.080
20	9	0.072
21	3	0.024
22	1	0.008
23	4	0.032
26	6	0.048
27	3	0.024
28	5	0.040
29	1	0.008
30	3	0.024
September:		
3	14	0.112
4	6	0.048
5	7	0.056
6	3	0.024
9	10	0.080
10	7	0.056
11	5	0.040
12	0	0.000
13	3	0.024

a. Construct a p chart if you did not already do so for problem 6.8.
b. Is the process in a state of statistical control? If not, when is it out of control?
c. Construct an np chart.
d. Compare the results of the np chart with the results obtained with the p chart and explain any similarities or differences.

7.13 The following 32 days of data represent the findings from a study conducted at a factory that manufactures film canisters. Each day 500 film canisters were sampled and inspected. The number of defective film canisters (non-conforming items) were recorded each day as follows.

CANISTER

Day	# Non-Conforming
1	26
2	25
3	23
4	24
5	26
6	20
7	21
8	27
9	23
10	25
11	22
12	26
13	25
14	29
15	20
16	19
17	23
18	19
19	18
20	27
21	28
22	24
23	26
24	23
25	27
26	28
27	24
28	22
29	20
30	25
31	27
32	19

a. Construct an np control chart using the first 25 data points to calculate trial limits.
b. Is the process in a state of statistical control? If not, when it is out of control?

c. If the process is in control, extend the limits and record data for days 26 through 32.
d. Is the process in control for days 26 through 32? If not, when it is out of control?
e. Construct a p chart using the first 25 data points if you did not already do so in problem 6.9.
f. Compare the results of the np chart with the results obtained with the p chart and explain any similarities or differences for the first 25 days.

7.14 The information systems department of a hospital is concerned with the time it takes for patients' medical records to be processed after discharge. They determine that all records should be processed within five days; any record not processed within five days of discharge is considered nonconforming. The number of patients discharged and the number and proportion of records not processed within the five day standard are recorded.

MEDREC

Day	Number of Discharges	Number of Medical Records Not Processed within 5 Days	Proportion of Non-Conforming Items
1	54	13	0.241
2	63	23	0.365
3	110	38	0.345
4	105	35	0.333
5	131	40	0.305
6	137	44	0.321
7	80	16	0.200
8	63	21	0.333
9	75	18	0.240
10	92	24	0.261
11	105	27	0.257
12	112	43	0.384
13	120	25	0.208
14	95	21	0.221
15	72	11	0.153
16	128	24	0.188
17	126	33	0.262
18	106	38	0.358
19	129	39	0.302
20	136	74	0.544
21	94	31	0.330
22	74	15	0.203
23	107	45	0.421
24	135	53	0.393
25	124	57	0.460
26	113	28	0.248
27	140	38	0.271
28	83	21	0.253
29	62	10	0.161
30	106	45	0.425

a. Construct an appropriate control chart for these data.
b. Is the process in a state of statistical control? Why?
c. If the process is not in a state of statistical control, eliminate out of control points and recalculate the trial control limits.

7.15 A company fills bulk orders of electronic telephones and is concerned about the number of units that were returned. As part of their investigation of the problem they sample orders and record order size and the number of telephones returned.

PHONES

Order	Order Size	Number Returned	Fraction Returned
1	350	12	0.034
2	420	29	0.069
3	384	23	0.060
4	840	33	0.039
5	405	20	0.049
6	752	40	0.053
7	409	13	0.032
8	385	28	0.073
9	780	24	0.031
10	820	46	0.056
11	392	25	0.064
12	818	24	0.029
13	399	23	0.058
14	355	21	0.059
15	414	22	0.053
16	754	44	0.058
17	366	24	0.066
18	839	34	0.041
19	411	28	0.068
20	387	26	0.067
21	353	18	0.051
22	415	28	0.068
23	390	17	0.044
24	358	28	0.078
25	411	22	0.053

a. Construct an appropriate control chart for these data.
b. Is the process in a state of statistical control? If not, which orders are out of control?

7.16 A large metropolitan hospital provides laboratory services to physicians in the community. A physician who submits a specimen for analysis must fill out a form indicating the services requested, the types of analyses requested, and billing information. These lab slips also contain demographic information on the patient. Incomplete slips must be returned and resubmitted. This process is costly and may increase the time required to complete analyses. In an effort to establish whether the process is in a state of statistical control, data are collected for a 30 day period. The number of lab slips, the number of slips missing demographic information, and the fraction of incomplete slips are shown below.

LABSLIP

Date	Number of Lab Slips Received	Number of Lab Slips Missing Demographics	Fraction Incomplete Lab Slips
September:			
16	187	11	0.0588
17	216	15	0.0694
18	144	9	0.0625
19	166	7	0.0422
20	192	16	0.0833
23	158	10	0.0633
24	146	9	0.0616
25	199	7	0.0352
26	221	10	0.0452
27	159	4	0.0252
30	222	6	0.0270
October:			
1	230	16	0.0696
2	214	15	0.0701
3	198	8	0.0404
4	147	8	0.0544
7	159	7	0.0440
8	145	4	0.0276
9	202	8	0.0396
10	217	11	0.0507
11	204	16	0.0784
14	229	13	0.0568
15	219	8	0.0365
16	211	5	0.0237
17	154	9	0.0584
18	188	13	0.0691
21	146	7	0.0479
22	172	12	0.0698
23	158	7	0.0443
24	148	6	0.0405
25	190	8	0.0421

a. Construct an appropriate control chart for these data.
b. Is the process in a state of statistical control? If not, on which dates is it out of control?

7.17 A commuter railroad in a large northeastern city runs 122 trains from suburban areas into the city each weekday. A survey of rider satisfaction indicates that commuters are very concerned with trains arriving on time. Before making changes to the system to increase the proportion of on-time arrivals, the railroad wants to know whether the proportion of on-time arrivals is in a state of statistical control. The number of late trains for 30 weekdays is shown below.

RRLATE

Day	Number Late	Day	Number Late
1	3	16	7
2	1	17	3
3	1	18	4
4	4	19	7
5	5	20	5
6	4	21	2
7	6	22	6
8	3	23	2
9	4	24	4
10	5	25	4
11	6	26	5
12	1	27	4
13	7	28	6
14	4	29	1
15	4	30	2

a. Construct an appropriate control chart for these data if you have not already done so in problem 6.11.
b. Is the process in a state of statistical control? If not, on which days is it out of control?
c. Construct an alternative control chart for these data.
d. Is the process in a state of statistical control? If not, on which days is it out of control?

7.18 The management of a city rapid transit system is concerned about the number of accidents reported and would like to know if the number of accidents is in a state of statistical control before instituting changes in procedure. The number of accidents reported each week for a 52 week period is shown below.

ACCIDENTS

Month	Week	Number of Accidents
January	1	175
	2	111
	3	77
	4	106
February	5	116
	6	57
	7	119
	8	109
March	9	106
	10	128
	11	104
	12	107
April	13	113
	14	99
	15	119
	16	99
May	17	112
	18	99
	19	76
	20	88
	21	76
June	22	98
	23	109
	24	100
	25	85
July	26	134
	27	141
	28	98
	29	55
	30	85
August	31	101
	32	67
	33	98
	34	96
September	35	94
	36	82
	37	135
	38	95
October	39	86
	40	73
	41	101
	42	113
	43	124

November	44	110
	45	124
	46	108
	47	81
December	48	93
	49	111
	50	123
	51	103
	52	169

a. Construct a control chart for these data.
b. Is the process in a state of statistical control? If not, when is the process out of control?

7.19 The manager of a regional office of a telephone company has the responsibility of processing requests for additions, changes, or deletions of telephone service. A service improvement team studies such orders in terms of central office equipment and facilities required to process orders. They find that errors requiring correction should be reduced. Before suggesting changes in the process, they monitor the number of errors to determine whether or not the system is stable. Their data collected over a 30 day period are shown below.

CORRECT

Day	Number of Orders	Number Corrections	Day	Number of Orders	Number Corrections
1	600	80	16	831	91
2	676	88	17	816	80
3	896	74	18	701	96
4	707	94	19	761	78
5	694	70	20	851	85
6	765	95	21	678	65
7	788	73	22	915	74
8	794	103	23	698	68
9	694	100	24	821	72
10	784	103	25	750	101
11	812	70	26	600	91
12	759	83	27	744	64
13	781	64	28	698	67
14	682	64	29	820	105
15	802	72	30	732	112

a. Construct a control chart for these data.
b. Is the process in a state of statistical control? If not, on which days is it out of control?

7.20 A private mail delivery service has a policy of guaranteeing delivery by 10:30 A.M. of the morning after a package is picked up. Suppose that management wishes to study delivery performance in a particular geographic area over a four-week time period based on a five-day workweek. The total number of packages delivered daily and the number of packages that were not delivered by 10:30 A.M. are recorded. The results are shown below.

MAILSPEC

Day	Packages Delivered	Packages Not Arriving Before 10:30 A.M.	Day	Packages Delivered	Packages Not Arriving Before 10:30 A.M.
1	136	4	11	157	6
2	153	6	12	150	9
3	127	2	13	142	8
4	157	7	14	137	10
5	144	5	15	147	8
6	122	5	16	132	7
7	154	6	17	136	6
8	132	3	18	137	7
9	160	8	19	153	11
10	142	7	20	141	7

a. Set up an appropriate control chart for the proportion of packages that are not delivered before 10:30 A.M.
b. Does the process give an out-of-control signal?

7.21 The owner of a dry-cleaning business, in an effort to measure the quality of the services provided, would like to study the number of dry-cleaned items that are returned for rework per day. Records are kept for a four-week period (the store is open Monday–Saturday) with the following results.

DRYCLEAN

Day	Items Returned for Rework	Day	Items Returned for Rework
1	4	13	5
2	6	14	8
3	3	15	3
4	7	16	4
5	6	17	10
6	8	18	9
7	6	19	6
8	4	20	5
9	8	21	8
10	6	22	6
11	5	23	7
12	12	24	9

a. Construct an appropriate control chart for the number of items per day that are returned for rework.
b. Is the process is in a state of statistical control?
c. Should the owner of the dry-cleaning store take action to investigate why 12 items were returned for rework on day 12? Explain. Would your answer be the same if 20 items were returned for rework on day 12?

7.22 The branch manager of a savings bank has recorded the number of errors of a particular type that each of 12 tellers has made during the past year. The results are shown below.

TELLER

Teller	Number of Errors
Anita	4
Carla	7
George	12
Jed	6
Linda	2
Matthew	5
Mitchell	6
Ned	3
Ron	5
Susan	4
Tamara	7
Victor	5

a. Do you think the bank manager will single out George for any disciplinary action regarding his performance in the last year? Why?
b. Construct an appropriate control chart for the number of errors committed by the 12 tellers. Is the number of errors in a state of statistical control?
c. Based on the control chart developed in (b), do you now think that George should be singled out for special attention regarding his performance? Does your conclusion agree with what you expected the manager to do in part (a)?

7.23 Falls are one source of preventable hospital injury. Although most patients who fall are not hurt, a risk of serious injury is involved. The following data represent the number of patient falls per month over a 28-month period in a 19-bed AIDS unit at a major metropolitan hospital.

PTFALLS

Month	Number of Patient Falls	Month	Number of Patient Falls
1	2	15	6
2	4	16	5
3	2	17	3
4	4	18	8
5	3	19	6
6	3	20	3
7	1	21	9
8	4	22	4
9	5	23	5
10	11	24	0
11	8	25	2
12	7	26	6
13	9	27	5
14	10	28	7

a. Construct an appropriate control chart for the number of patient falls per month.
b. Is the process of patient falls per month in a state of statistical control?
c. If not, during which months is it out of control?

7.24 The funds transfer research department of a bank is concerned with turnaround time for investigations of funds-transfer payments. A payment may involve the bank as a remitter of funds, a beneficiary of funds, or an intermediary in the payment. An investigation is initiated by a payment inquiry or query by a party involved in the payment or any department affected by the flow of funds. Once a query is received, an investigator reconstructs the transaction trail of the payment and verifies that the information is correct and the proper payment is transmitted. The investigator then reports the results of the investigation and the transaction is considered closed. It is important that investigations are closed rapidly, preferably within the same day. The number of new investigations and the number and proportion closed on the same day that the inquiry was made are shown below.

FUNDTRAN

Day	New Investigations	Number Closed
1	240	96
2	296	88
3	309	113
4	293	138
5	253	119
6	254	94
7	245	75
8	331	125
9	303	134
10	278	83
11	256	90
12	273	102
13	276	115
14	291	98
15	204	83
16	263	79
17	311	116
18	248	104
19	287	110
20	238	107
21	280	131
22	271	139
23	237	121
24	258	94
25	289	128
26	226	90
27	287	106
28	263	81
29	282	107
30	194	75

a. Construct an appropriate control chart for these data.
b. Is the process in statistical control? If not, on which days is it out of control?

7.25 The manager of a retail sales branch of a brokerage office is concerned with the number of undesirable trades made by the sales staff. A trade is considered undesirable if there is an error on the trade ticket. Trades that are in error must to be cancelled and resubmitted. The cost of correcting errors is billed to the brokerage. In studying the problem, the manager wants to know whether the proportion of undesirable trades is in a state of statistical control so he can plan the next step in a quality improvement process. Data is collected for a 30-day period with the following results.

TRADE

Day	Undesirable Trades	Total Trades
1	2	74
2	12	85
3	13	114
4	33	136
5	5	97
6	20	115
7	17	108
8	10	76
9	8	69
10	18	98
11	3	104
12	12	98
13	15	105
14	6	98
15	21	204
16	3	54
17	12	74
18	11	103
19	11	100
20	14	88
21	4	58
22	10	69
23	19	135
24	1	67
25	11	77
26	12	88
27	4	66
28	11	72
29	13	118
30	15	138

a. Construct an appropriate control chart for these data.

b. Is the process in a state of statistical control? If not, on which days is it out of control?

7.26 Rochester-Electro-Medical Inc. is a manufacturing company based in Tampa, Florida that produces medical products. Recently, management felt the need to improve the safety of the workplace and began a safety sampling study. The data that follows represents the number of unsafe acts observed by the company safety director over an initial time period in which the study was carried out.

SAFETY

Tour	Number of Unsafe Acts
1	10
2	6
3	6
4	10
5	8
6	12
7	2
8	1
9	23
10	3
11	2
12	8
13	7
14	6
15	6
16	11
17	13
18	9
19	6
20	9

Source: H. Gitlow, A. R. Berkins, and M. He, " Safety Sampling: A Case Study," *Quality Engineering*, 14, 2002, p. 405 – 419

a. Construct an appropriate control chart for the number of unsafe acts.
b. Based on the results of (a), is the process in a state of statistical control?
c. If not, when is the process out of control?

REFERENCES AND ADDITIONAL READINGS

[1] American Society for Quality Control Inc., 310 West Wisconsin Ave., Milwaukee, Wis. 53203.

[2] Gitlow, H. and Gitlow, S.(1987), The Deming Guide to Quality, Productivity and Competitive Position, Prentice-Hall Publishers (Englewood Cliffs, N.J.).

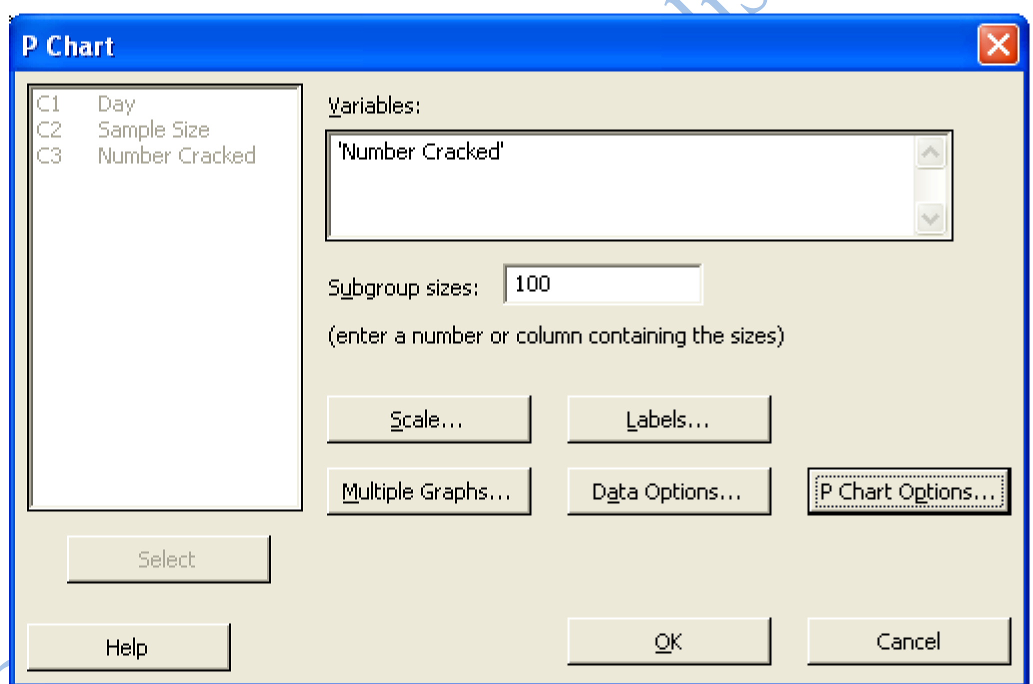
Appendix A7.1

Using Minitab for Attribute Charts

Using Minitab for the p Chart

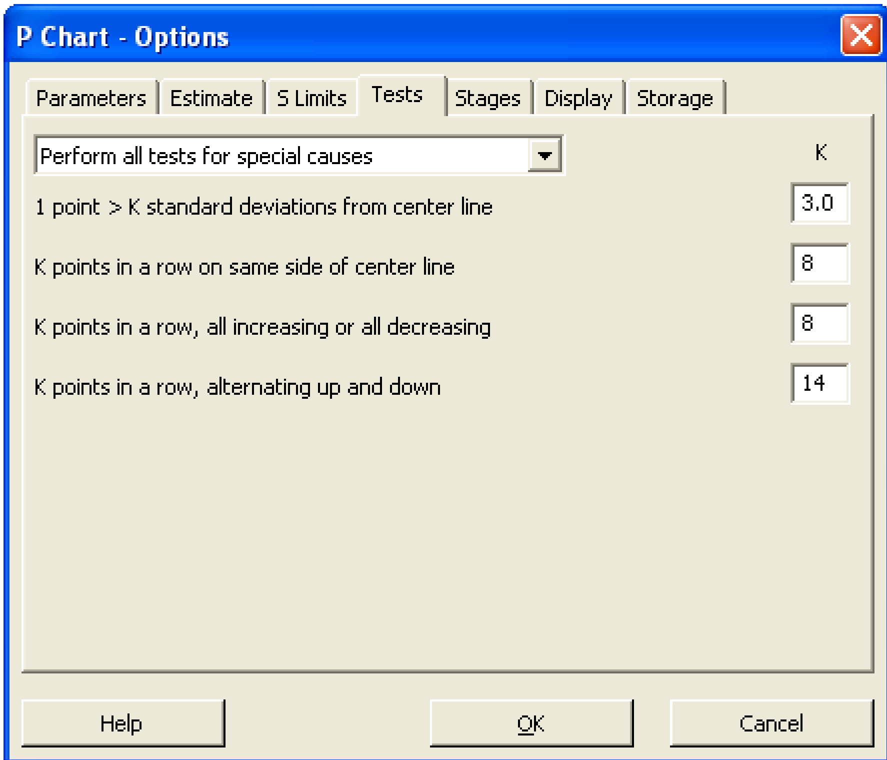
To illustrate how to obtain a p chart, refer to the data of Table 7.1 concerning the number of broken tiles. Open the **TILES.MTW** worksheet. TILES. Select **Stat | Control Charts | Attribute Charts | P**. In the P Chart dialog box, shown in Figure A7.1, enter **C3** or **'Number Cracked'** in the Variable edit box. Since the subgroup sizes are equal, enter **100** in the Subgroup Size box. Select the **P Chart Options** button.

Figure A7.1
Minitab p Chart Dialog Box



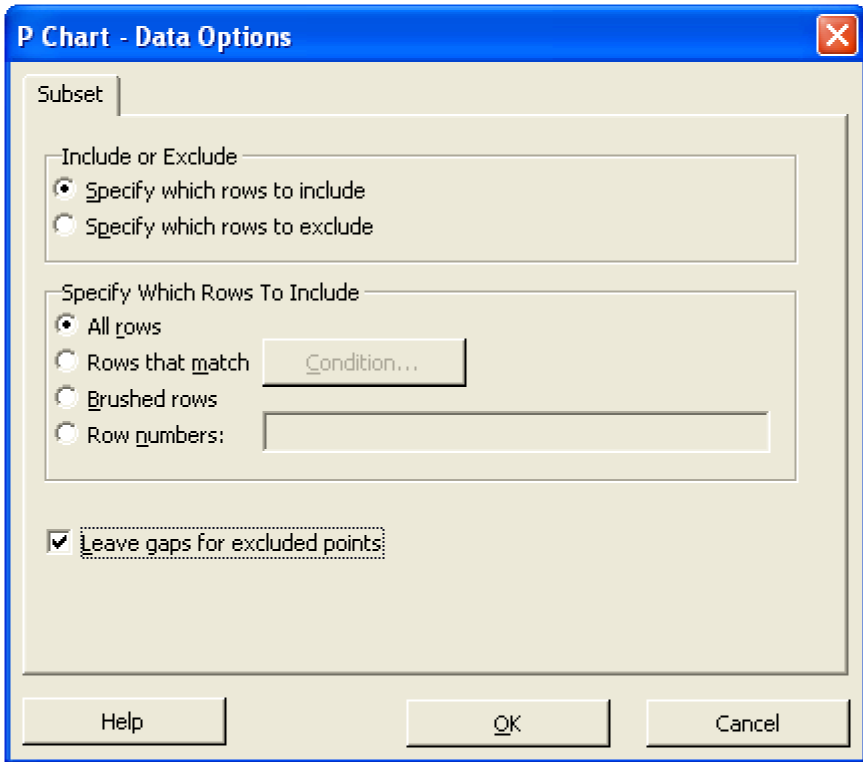
1. In the P Chart Options dialog box, click on the Tests tab, shown in Figure A7.2. Select all of the tests and enter the values you would like to use (here we have conformed to the Chapter 6 rules) as shown in Figure A7.2. Click the **OK** button to return to the P Chart dialog box. These values will stay intact until Minitab is restarted.

Figure A7.2
Minitab P Chart Options



2. If there are points that should be omitted when estimating the centerline and control limits, click the **Data Options** tab in the P Chart Options dialog box, as shown in Figure A7.1. The resulting screen will appear as in Figure 7.3. Enter the points to be omitted in the proper box. Click the **OK** button to return to the P Chart dialog box. In the P Chart dialog box, click the **OK** button to obtain the p chart.

Figure A7.3
Minitab p Chart-Options Dialog Box, Estimate Tab



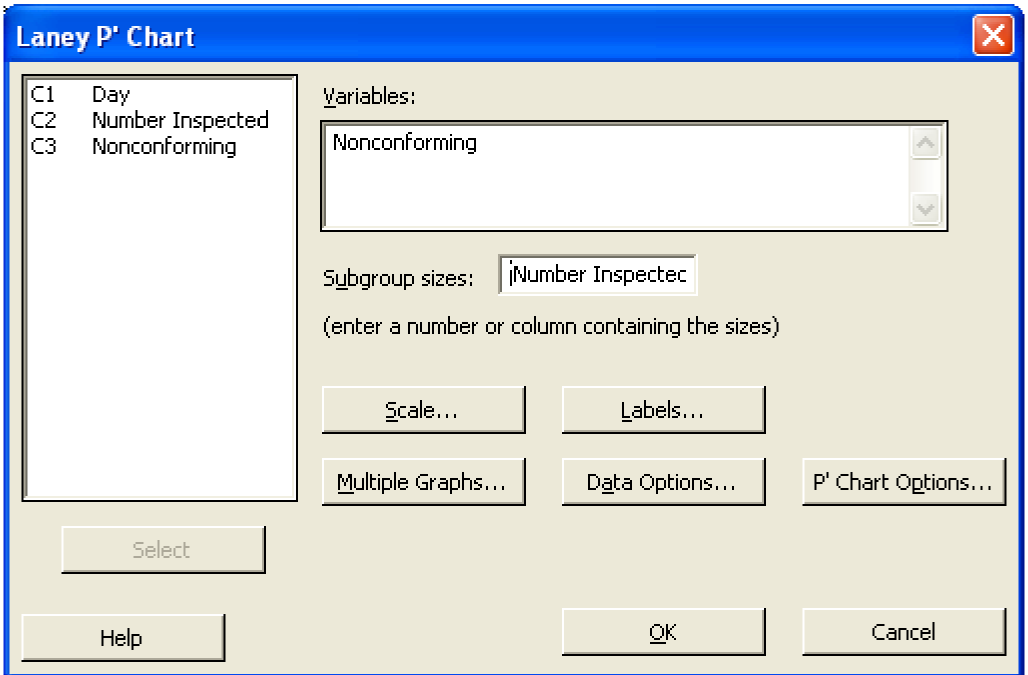
Minitab p Chart Dialog Box for Unequal Sample Sizes

To illustrate an example in which the subgroups sizes differ, open the **INSULATOR.MTW** file . Select **Stat | Control Charts | Attribute Charts | Laney P Chart**

1. In the dialog box, shown in Figure A7.4, enter **C3** or **Nonconforming** in the Variable edit box.

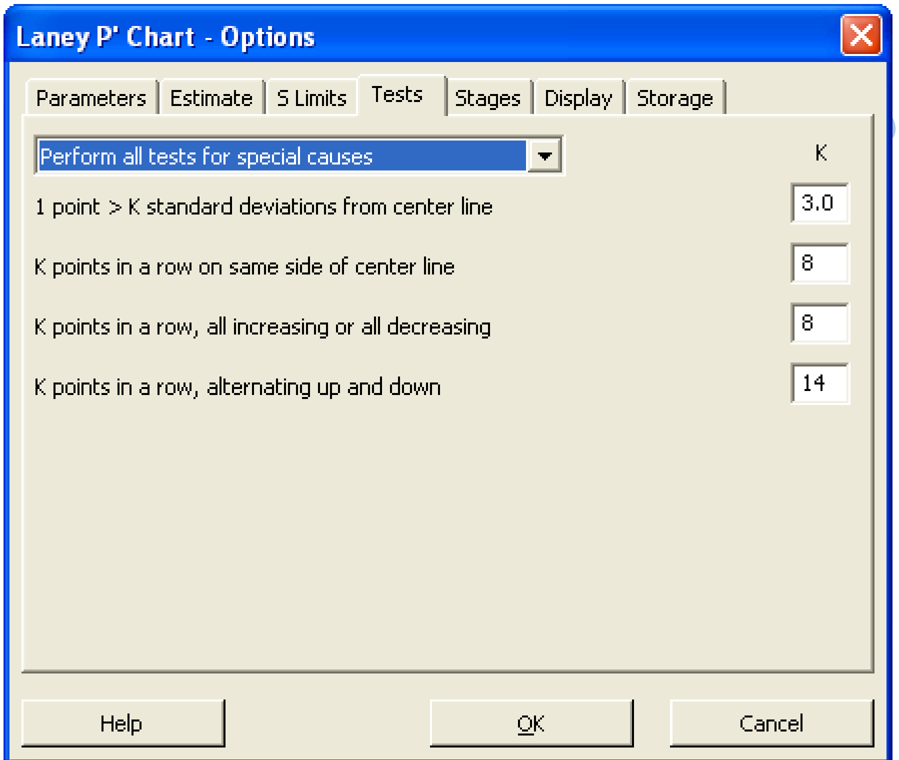
2. Enter or select Number Inspected in the **Subgroup sizes** box

Figure A7.4
Laney p Chart Dialog Box



3. Select the **Laney P Chart Options** button. In the P Chart Options dialog box, click on the Tests tab, as shown in Figure A7.2. Select the all tests with the appropriate values entered as in Figure A7.5. Click the **OK** button to return to the P Chart dialog box.

Figure A7.5
Laney p chart Options Dialog Box – Tests



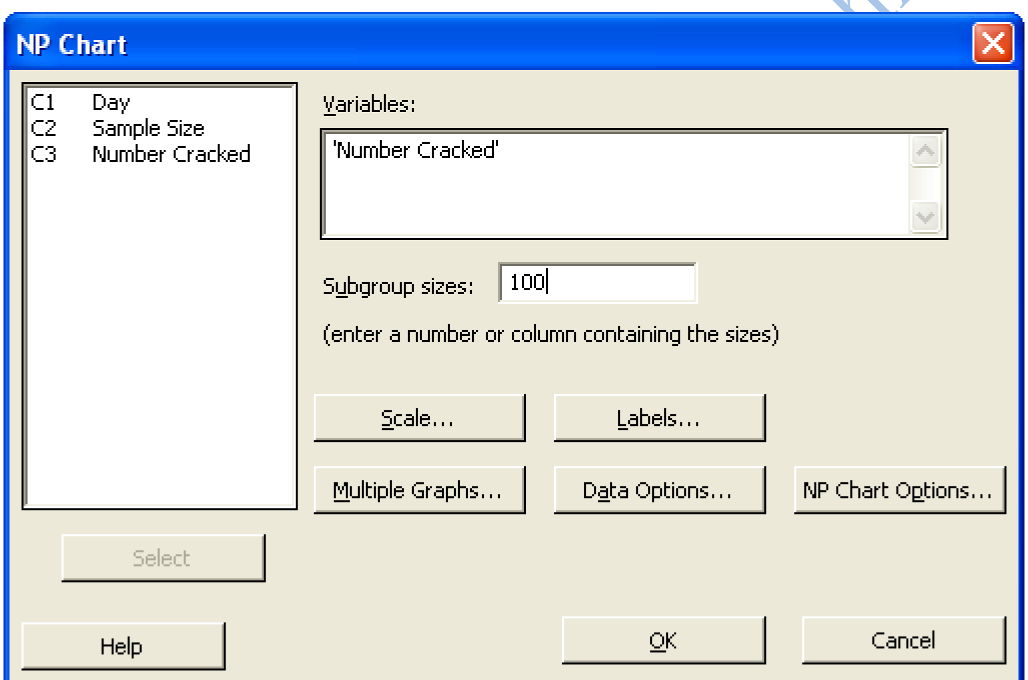
Using Minitab for the np Chart

To illustrate how to obtain an np chart, refer to the data of Table 7.1 on page 000 concerning the number of cracked tiles. Open the **TILES.MTW** worksheet .

1. Select **Stat | Control Charts | Attribute Charts | NP**. In the NP Chart dialog box, shown in Figure A7.6, enter **C3** or **'Number Cracked'** in the Variable edit box. Select **Size** in the **Subgroup** drop-down list box and enter **100** in the edit box. Select the **NP Chart Options** button.

2. In the NP Chart Options dialog box, click on the Tests tab. Select all the tests with appropriate values specified, and then click the **OK** button to return to the NP Chart dialog box. These values will stay intact until Minitab is restarted.
3. If there are points that should be omitted when estimating the centerline and control limits, click the **Estimate** tab in the NP Chart Options dialog box. Enter the points to be omitted in the edit box shown. Click the **OK** button to return to the P Chart dialog box. In the P Chart dialog box, click the **OK** button to obtain the p chart.

Figure A7.6
Minitab np Chart Dialog Box



Using Minitab for the c Chart

To illustrate how to obtain a c chart, refer to the data of Table 7.7 on page 000 concerning the number of blemishes in reels of paper. Open the **REEL.MTW** worksheet  .

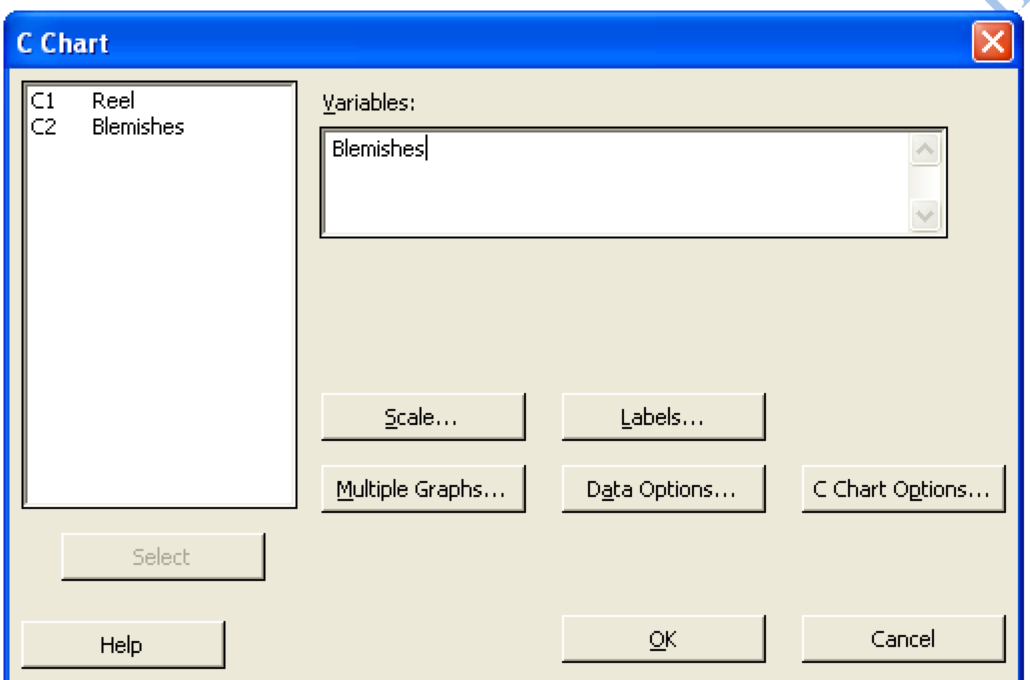
Select **Stat | Control Charts | Attribute Charts | C**. Enter **C2** or **BLEMISHES** in the Variable edit box.

1. In the c Chart Options dialog box, shown in Figure A7.7, click on the Tests tab. Select the **perform all tests** option button. Click the **OK** button to return to the c Chart dialog box.

These values will stay intact until Minitab is restarted. In the c Chart dialog box, click the **OK** button to obtain the c chart.

2. If there are points that should be omitted when estimating the centerline and control limits, click the **Estimate** tab in the c Chart Options dialog box. Enter the points to be omitted in the edit box shown. Click the **OK** button to return to the c Chart dialog box.

Figure A7.7
Minitab c Chart Dialog Box



Using Minitab for the u Chart

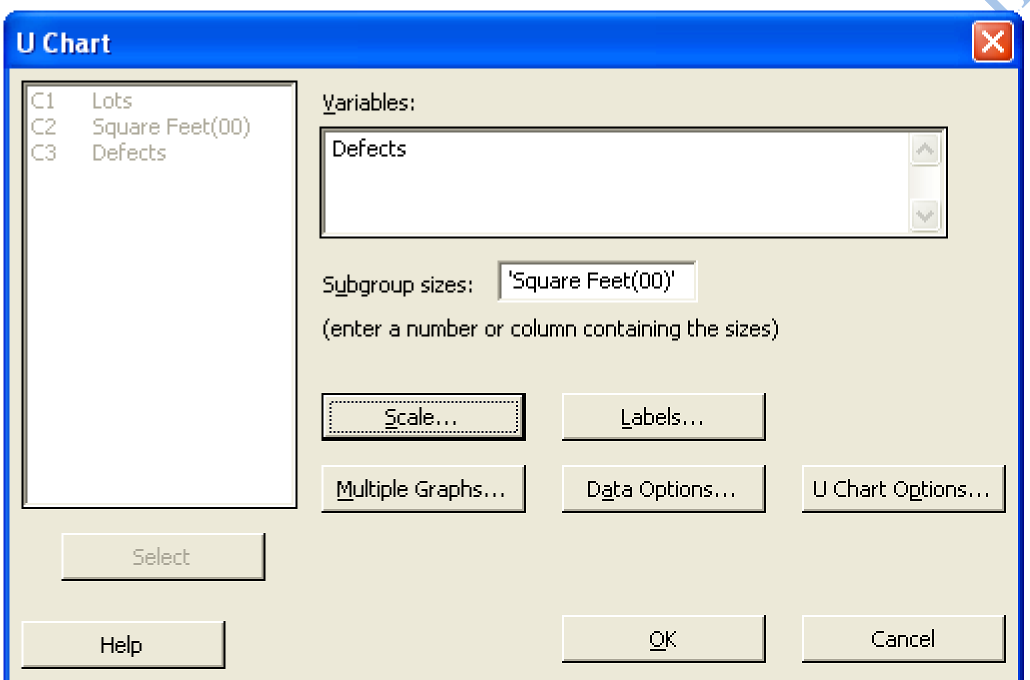
To illustrate how to obtain a u chart, refer to the data of Table 7.13 page 000 concerning the number of defects in a lot of plastic. Open the **PLASTIC.MTW** worksheet.

1. Select **Stat | Control Charts | Attribute Charts | U**. Enter **C3** or **DEFECTS** in the Variable edit box. In the Subgroups: drop-down list box, select Indicator column: and enter **C2** or **'Square Feet (00)'** in the edit box.
2. In the U Chart Options dialog box, shown in Figure A7.8, click on the Tests tab. Select all the tests with appropriate values, and then click the **OK** button to return to the U Chart

dialog box. These values will stay intact until Minitab is restarted. In the U Chart dialog box, click the **OK** button to obtain the u chart.

3. If there are points that should be omitted when estimating the centerline and control limits, click the **Estimate** tab in the U Chart Options dialog box. Enter the points to be omitted in the edit box shown. Click the **OK** button to return to the U Chart dialog box.

Figure A7.8
Minitab u Chart Dialog Box



Using Minitab to Obtain Zone Limits

To plot zone limits on any of the control charts discussed in this appendix, open to the Data Source dialog box for the control chart being developed and do the following:

1. Click the **Scale** button. Click the **Gridlines** tab. Select the **Y major ticks**, **Y minor ticks**, and **X major ticks** check boxes. Click the **OK** button to return to the Data Source dialog box.

2. Select the **Options** button. Select the **S limits** tab. In the Standard deviation limit positions: edit box, select **Constants** in the drop-down list box and enter **1 2 3** in the edit box. Click the **OK** button to return to the Data Source dialog box.