CHAPTER 13
INSPECTION POLICY

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Chapter Objectives

- To introduce three alternative inspection policies: no inspection; 100% inspection; and sampling inspection, or acceptance sampling
- To discuss three classes of acceptance sampling plans: lot-by-lot acceptance sampling; continuous flow acceptance sampling; and special sampling plans
- To discuss the disadvantages of using acceptance sampling
- To demonstrate the lack of validity of acceptance sampling plans for stable processes
- To discuss and illustrate the kp rule; an optimal inspection policy for stable processes
- To discuss exceptions to the kp rule, when testing is destructive or the product tested is a homogeneous mixture
- To discuss inspection policies for unstable processes, for the cases of mild chaos and severe chaos
- To discuss exceptions to the rules for inspecting unstable processes

13.1 Introduction

This chapter introduces policies and procedures for inspection of incoming, intermediate, and final goods and services. Three options exist for inspection of goods and services: (1) no inspection, (2) 100 percent inspection, and (3) sampling inspection. The first part of the chapter focuses on sampling inspection, commonly called acceptance sampling, as a
method to determine whether to accept, reject, or screen goods and services. Three types of acceptance sampling plans are discussed: lot-by-lot plans, continuous flow plans, and special sampling plans. The second part of the chapter presents a theoretical argument against using acceptance sampling plans, followed by a discussion of W. Edwards Deming’s “kp rule,” an alternative inspection procedure that minimizes the total cost of inspection for incoming, intermediate, and final goods and services. The chapter ends with two mathematical proofs of the arguments made.

13.2 Inspecting Goods and Services

Goods or services enter an organization from a vendor or are passed on internally from one part of the organization to another, such as department to department, or operation to operation within a department. These goods or services move inter- or intra-organizationally either in discrete lots or in continuous flows, and have certain customer-specified quality characteristics.

Organizations, or their subcomponents, require some method for minimizing the total cost of inspecting incoming and intermediate goods or services, plus the cost to repair and test these goods and services in process; or, of inspecting final goods or services that fail to meet specifications because of a defective good or service used in production. The three alternatives for inspection of goods or services are:

1. No inspection (send items straight into use with no screening),
2. 100 percent inspection (screen all goods or services to weed out defectives), or
3. Sampling inspection, also known as acceptance sampling (screen a sample of goods or services to determine if the remainder should be accepted, rejected, or screened).

Historically, acceptance sampling has been considered useful if the inspection process is destructive (100 percent inspection will destroy all goods or services), the cost of 100 percent inspection is high, or too many units have to be inspected.

13.3 Acceptance Sampling

The purpose of acceptance sampling is to determine the disposition of goods or services: accept, reject, or screen. We wish to select the disposition that minimizes the cost of inspection to achieve a desired level of quality (called the Acceptable Quality Level or AQL, as discussed in Chapter 11). Table 13.1 shows several types of acceptance sampling plans [Duncan, 1986, pp. 161-414].
### Table 13.1
Acceptance Sampling Plans

<table>
<thead>
<tr>
<th>Types of Plans</th>
<th>Variables</th>
<th>Accept/Reject</th>
<th>Rectifying</th>
<th>Attributes</th>
<th>Accept/Reject</th>
<th>Rectifying</th>
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<tbody>
<tr>
<td>Lot-by-Lot</td>
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<tr>
<td>Continuous Flow</td>
<td>Attributes</td>
<td></td>
<td>Rectifying</td>
<td></td>
<td></td>
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<tr>
<td>Special Sampling Plans</td>
<td>Skip-Lot</td>
<td></td>
<td>Rectifying</td>
<td>Variables</td>
<td></td>
<td>Narrow Limit</td>
</tr>
</tbody>
</table>

#### 13.3.1 Lot-by-Lot Acceptance Sampling

Lot-by-lot acceptance sampling plans are used to inspect goods or services whenever the goods or services can conveniently be grouped into lots. All lot-by-lot acceptance plans are based on accepting, rejecting, or screening the remainder of the lot based on the number of defects found in the sample. Lot-by-lot plans can be designed for both attribute data and variables data. Such plans can also be broken down into acceptance/rejection plans (plans in which a sample of items is drawn from the lot and the remainder of the lot is accepted or rejected based on an analysis of the sample) or rectifying plans (plans that call for either total or partial screening of the remainder). The most common acceptance/rejection lot-by-lot acceptance sampling plan for variables used in American industry today is Military Standard 414 [Duncan, 1986, pp. 291-305]. This plan is used to control the fraction of incoming material that does not conform to specifications for variables data. The most common acceptance/rejection lot-by-lot acceptance sampling plan for attributes is Military Standard 105D [Duncan, 1986, pp. 217-48]. This plan is used to constrain suppliers so that they will deliver at least an Acceptable Quality Level of goods or services for attribute data.

#### 13.3.2 Continuous Flow Acceptance Sampling

Continuous flow plans are used to inspect goods or services whenever the goods or services cannot be grouped into lots -- for example, goods on conveyor belts or goods on a continuous moving line. Only rectifying attribute sampling plans exist for continuous flow processes. The most common acceptance sampling plan in this category used today is
Military Standard 1235B [Duncan, 1986, pp. 406-13]. All continuous flow acceptance plans are based on a clearance number, the number of conforming units observed between the occurrences of two defective units. If the number of units between two defective units is greater than the number of units specified in the clearing sample, the units will be accepted and shipped; otherwise, they will be 100 percent inspected.

13.3.3 Special Sampling Plans

Other types of acceptance sampling plans have been developed that are of the lot-by-lot type, but are applied to a series of lots considered as a group. They are called special sampling plans. These plans include control chart plans [Duncan, 1986, pp. 536-40], chain sampling plans [Duncan, 1986, pp. 177-79], skip-lot plans [Duncan, 1986, pp. 252-53], variables plans [Duncan, 1986, pp. 340-65], and narrow limit plans [Duncan, 1986, pp. 271-72]. Special sampling plans are not discussed in this book.

Much attention is given to acceptance sampling plans in many textbooks and courses. However, these plans do not minimize the total cost of inspection of: (1) incoming and intermediary goods or services, plus the cost to repair and test these goods and services in process, or (2) final goods or services that fail to meet specifications because of a defective good or service that was used in production. They also emphasize inspection, not process improvement which can eliminate inspection. Furthermore, there is a strong theoretical basis for not using acceptance sampling plans, as we demonstrate below.

13.4 A Theoretical Invalidcation of Acceptance Sampling

A discussion of the lack of validity of acceptance sampling must consider the stability, or lack of stability, of the process's output which is to undergo inspection. Let us first consider the case of a stable process.

13.4.1 Stable Process

Suppose a lot of N independent items are drawn from a stable process that generates 100p percent defective output, and that x of the items are defective and N - x are conforming. Here the number of defectives is binomially distributed with fraction defective p. This is extremely common in stable processes. Then

\[ N = \text{total number of items in the lot}, \]
\[ x = \text{number of defective items in the lot}, \]
\[ N - x = \text{number of conforming items in the lot}, \text{ and} \]
\[ E(x/N) = p = \text{fraction defective items in the process}. \]

Suppose a sample of n items is drawn from the lot of N items (without replacement, because of the finite nature of the lot), such that r of the items are defective and n - r of the items are conforming. Then

\[ n = \text{total number of items in the sample}, \]
\[ r = \text{number of defective items in the sample}, \]
\[ n - r = \text{number of conforming items in the sample}, \]
\[ E(r/n) = p = \text{estimated fraction defective items in the process}. \]

The selection of the sample from the lot creates a new entity we will call the remainder, or the rest, of the lot. The remainder is composed of \( N - n \) items, such that \( x - r \) of the items are defective and \( (N - n) - (x - r) \) are conforming. Then

\[ N - n = \text{total number of items in the remainder}, \]
\[ x - r = \text{number of defective items in the remainder}, \]
\[ (N - n) - (x - r) = \text{number of conforming items in the remainder}, \]
\[ E[(x - r)/(N - n)] = p = \text{estimated fraction defective items in the process}. \]

Figure 13.1 illustrates this sequence of item groupings.

**Figure 13.1**
Selection of Samples from Lots Drawn from a Process

If it can be shown that the number of defectives in the sample is independent of, or not correlated with, the number of defectives in the remainder, then acceptance sampling plans that determine the disposition of a remainder (accept, reject, or screen) based on the number of defectives in a sample are invalid; this proof appears in Appendix 13.1.

In other words, if the number of defectives in the sample and in the number of defectives in the remainder are both binomially distributed with the same mean fraction defective (\( p \)), then they are independent of each other. For example, if a lot of 1,000 fair coins was tossed repeatedly and a sample of 50 of the 1,000 was drawn each time for inspection, the fraction of heads in the sample and in the remainder would both be distributed around \( p = 0.5 \), but the number of heads in the sample would be independent of the number of
heads in the remainder. This means the distribution of heads in the remainder associated with samples yielding 0 heads would be the same as the distribution of heads in the remainder associated with samples yielding 50 heads. Hence, the results of the sample are not useful in predicting what would happen to the remainder; acceptance sampling plans that determine the disposition of the remainder based on samples are thus not valid for a stable process. This surprising result [Mood, 1943, pp. 415-25; Deming, 1950, p. 258] does not mean that statistical inference is invalid, that is, that a random sample from a population does not provide information about the frame. Rather, it says only that samples provide no information about remainders from stable processes. An alternative to acceptance sampling from stable processes must be found. W Edwards Deming offered, as an alternative, the kp rule, to be discussed later in this chapter.

13.4.2 Chaotic Process

Suppose a lot of N items is drawn from a chaotic or unknown process. In the lot, x of the items are defective and N - x are conforming, and the process fraction defective p varies from lot to lot (or day to day) and is not predictable. Then

\[
\begin{align*}
N &= \text{total number of items in the lot}, \\
x &= \text{number of defective items in the lot, and} \\
N - x &= \text{number of conforming items in the lot.}
\end{align*}
\]

Suppose a sample of n items is drawn from the lot of N items (without replacement, because of the finite nature of the lot) such that r of the items are defective and n - r of the items are conforming. Then

\[
\begin{align*}
n &= \text{total number of items in the sample}, \\
r &= \text{number of defective items in the sample, and} \\
n - r &= \text{number of conforming items in the sample.}
\end{align*}
\]

As with the stable process, the selection of the sample from the lot creates a remainder composed of N - n items, such that x - r of the items are defective and (N - n) - (x - r) are conforming. Then

\[
\begin{align*}
N - n &= \text{total number of items in the remainder,} \\
x - r &= \text{number of defective items in the remainder, and} \\
(N - n) - (x - r) &= \text{number of conforming items in the remainder.}
\end{align*}
\]

As before, Figure 13.2 represents the sequence of item groupings discussed in this section.

Here we can show that the number of defectives in the sample, r, is correlated with the number of defectives in the remainder, x - r. Thus, when p varies widely and unpredictably from lot to lot, the information from a sample provides insight into the remainder. Going back to the coin example, if lots of 1,000 biased coins were tossed repeatedly and samples of 50 of the 1,000 were drawn each time for inspection, the distribution of the
fraction of heads in the samples and in the remainders would not be distributed around 0.5; rather, they would be related to the actual fraction of heads in the lot and would be correlated. Similarly, the number of defectives in the sample and remainder would be correlated. Hence, acceptance sampling plans that determine the disposition of a remainder (accept, reject, or screen) based on the number of defectives in a sample are valid for a chaotic process. Later in this chapter, we will discuss the larger question of whether it is the most cost-effective plan given the chaotic nature of the process.

Note that as processes are stabilized as a result of quality efforts, acceptance plans that are valid for chaotic processes, albeit at high cost, will no longer be effective on the stable process.

13.5 The kp Rule for Stable Processes

Given a stable process and the knowledge that acceptance sampling plans are not effective on stable processes, we are left with only two of the inspection alternatives discussed at the beginning of this chapter: no inspection or 100 percent inspection. We discuss here Deming's kp rule, which specifies when to do no inspection and when to do 100 percent inspection so as to minimize the total cost of incoming and intermediate materials, final products, and repairing and testing those products that fail. The rule is derived in Appendix 13.2.

The assumptions for the use of the kp rule are not restrictive and are applicable to many common situations [Deming, 1982, pp. 237-311; Orsini, 1982; Papadakis, 1985, pp. 121-127].

1. All items are tested (inspected) before they move forward in the process. In other words, all nonconforming items are detected by a final inspection. When all items will not be subjected to a final inspection, the rule can be modified to reflect the possibility that a certain fraction of nonconforming parts, f, would be caught and the remaining fraction, 1 - f, would continue on into production or into the hands of customers [Deming, 1950, p. 123].

2. Inspection is completely reliable. If an item is defective, it will fail inspection. As Deming wrote, "A defective part is one that by definition will cause the assembly to fail. If a part declared defective at the start will not cause trouble further down the line, or with the customer, then you have not yet defined what you mean by a defective part" [Deming, 1950, p. 268].

3. The item vendor will give the buyer an extra supply of items, S, to replace any defective item found. The supplier adds the cost of these items onto her bill, either directly or indirectly. This cost is an overhead cost and would be present regardless of the inspection plan used. Hence, it need not be included in the cost function to be minimized.
The following notation is necessary to determine when to do 100 percent inspection and when to do no inspection:

\[ p = \text{the average incoming fraction of defective items in incoming lots of items.} \]

Recall that the process under study is stable and has a meaningful average incoming fraction of defective items, \( p \), in incoming lots of items;

\[ k_1 = \text{the cost to initially inspect one item;} \]

\[ k_2 = \text{the cost to dismantle, repair, reassemble, and test a good or service that fails because a defective item was used in its production.} \]

If a process is stable around the fraction \( p \), the kp rule states

1. If \( \frac{k_1}{k_2} \) is greater than \( p \), then 0 percent inspection. That is, no inspection is the policy that minimizes the total cost. This occurs if the fraction of incoming defective items, \( p \), is very low, the cost of inspecting an incoming item is high, and the cost of the defective item getting into production is low; therefore, no inspection is needed. The rationale is that there is little risk or penalty associated with incoming defective items.

2. If \( \frac{k_1}{k_2} \) is less than \( p \), then 100 percent inspection. That is, one hundred percent inspection is the policy that minimizes the total cost. This occurs if the fraction of incoming defective items, \( p \), is high, the cost of inspecting an incoming item is low, and the cost of the defective item getting into production is high; therefore 100 percent inspection is needed. The rationale here is that there is great risk and penalty attached to incoming defective items.

3. If \( \frac{k_1}{k_2} \) equals \( p \), then either 0 percent or 100 percent inspection. A decision must be made as to whether 0 percent or 100 percent inspection should be done in this case. In general, if \( p \) is not based on a substantial past history, 100 percent inspection is vital for safety’s sake.

To summarize, the kp rule will minimize the total cost of incoming and intermediary materials and final product for a stable process by proper selection of a 0 percent or 100 percent inspection policy. If the process under study is stable, then whether item \( i \) is defective is independent of whether any other item is defective. Hence, item \( i \) should be inspected, or not inspected, according to whether \( p \) is greater than or less than \( \frac{k_1}{k_2} \).

Recall that item \( i \) is a randomly selected item, and policy set for item \( i \) applies to any item. We can thus extend the policy for item \( i \) to all items in the lot. Consequently, either all or no items in the lot should be inspected, depending on whether \( p \) is greater than or less than the breakeven point, \( \frac{k_1}{k_2} \).

It is important to note that 0 percent inspection does not mean zero information. Small samples should always be drawn from every lot -- or on a skip-lot basis -- for information about the process under study. This information should be recorded on control charts to facilitate process improvement [Deming, 1950, p. 273]. The cost of these small samples is
The kp rule is appropriate between any two points in an organization's interdependent system of stakeholders, such as internally, in the vendor's processes, or between the firm and the vendor.

13.5.1 An Example of the kp Rule

A car manufacturer is deciding whether to purchase $25 million worth of equipment that would test engines purchased from vendors. The vendor's process is stable. The following figures have been determined:

- The inspection cost to screen out incoming defective engines is $50 per engine ($k_1 = $50).
- The cost for corrective action if a defective engine gets into production is $500 per defective engine ($k_2 = $500).
- On average, 1 in 150 incoming engines is defective ($p = 1/150 = 0.0067$).

Consequently, $k_1/k_2 = 50/500 = 0.1$. Note that 0.1 is greater than 0.0067. Therefore, $k_1/k_2$ is greater than $p$, and the correct course of action would be to do no initial inspection on incoming engines to achieve minimum total cost.

If no engines are inspected, the auto company would expect to incur the $500 cost in 1 out of 150 engines. This translates into an average corrective action cost of $3.33 per engine ($500 x 1/150). By eliminating initial inspection, the company would save $46.67 per engine ($50 - $3.33) on average. As the company purchases 4,000 engines per day, this translates into a daily savings of $186,680 (4,000 x $46.67), not including the savings of $25 million for testing equipment, interest on that money, and time freed up to work on improving quality! The next step in the pursuit of quality is for the auto company to work with its engine vendor to reduce the fraction of defective engines [Deming, 1950, pp. 276-77].

13.5.2 Exceptions to the kp Rule

Destructive Testing. The kp rule does not apply to destructive testing, in which an item is destroyed in conducting the test. The only solution in destructive testing is to achieve statistical control such that $p < k_1/k_2$, so that no inspection (other than routine small samples of the process) is the minimum cost policy. Note that achieving statistical control with $p < k_1/k_2$ is always the best solution regardless of whether the test is destructive or nondestructive [Deming, 1950, pp. 274-75].

Homogeneous Mixtures. The kp rule does not apply to homogeneous mixtures -- for example, a gallon sampled from a tank of brine. It does not matter whether we draw off the gallon from the top of the tank, or middle, or bottom. In this case, the sample is...
identical in composition to the remainder; hence, we can make judgments about the remainder from the sample [Deming, 1950, p. 285].

13.5.3 Component Costs of $k_1$ and $k_2$

Some costs to consider when calculating $k_1$ and $k_2$ are shown in Table 13.2 [Papadakis, 1985, p. 124]. The costs required to compute $k_1$ are usually known; a firm’s financial personnel should be helpful in computing $k_1$. However, the costs required to compute $k_2$ are generally unknown and frequently difficult to estimate (for example, the cost of customer dissatisfaction from recalls or lawsuits). A reasonable policy is to estimate $k_2$ without the more subjective costs. If $p > k_1/k_2$, there is no need to estimate the other components of $k_2$. On the other hand, if $p < k_1/k_2$ without including the subjective costs, an estimate of these missing costs may then have to be made to determine more accurately the relationship between $p$ and $k_1/k_2$.

Table 13.2
Component Costs of $k_1$ and $k_2$

<table>
<thead>
<tr>
<th><strong>Some Inspection Costs (k$_1$)</strong></th>
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<tbody>
<tr>
<td>Capital equipment</td>
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<tr>
<td>Initial cost</td>
<td></td>
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<tr>
<td>Depreciation (also considers residual value)</td>
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</tr>
<tr>
<td>Planned production volumes</td>
<td></td>
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<tr>
<td>Cost of capital</td>
<td></td>
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<tr>
<td>Operating costs</td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td></td>
</tr>
<tr>
<td>Rent, utilities, maintenance</td>
<td></td>
</tr>
<tr>
<td>Piece cost (outside vendor quote)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Possible Detrimental Costs (k$_2$)</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Added costs of processing the nonconforming item further</td>
<td></td>
</tr>
<tr>
<td>Cost of sorting lots later to find a nonconforming item</td>
<td></td>
</tr>
<tr>
<td>Cost of repairing batches of assemblies later</td>
<td></td>
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<tr>
<td>Cost of lost production later if lots of parts or batches of assemblies must be guaranteed pending sorting and repair</td>
<td></td>
</tr>
<tr>
<td>Warranty costs</td>
<td></td>
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<tr>
<td>Cost of recalls</td>
<td></td>
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<tr>
<td>Lawsuits ($k_2 \rightarrow \infty$ for safety items)</td>
<td></td>
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<tr>
<td>Customer loyalty impinging upon future sales</td>
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</table>

13.6 Inspection Policies for Chaotic Processes
Given a chaotic process and the knowledge that acceptance sampling plans are appropriate for such processes, we have the three inspection alternatives discussed earlier: (1) no inspection, (2) 100 percent inspection, and (3) some form of acceptance sampling. Our choice between alternatives (1) and (3) to achieve savings over 100 percent inspection (alternative (2)) depends on the nature of the chaos in the process.

13.6.1 Mild Chaos

If the fraction defective in the process under study wanders in an unpredictable manner so that the fractions for the worst lots are below \( k_1/k_2 \), as shown in Figure 13.2, then no inspection (except for routine sampling of the process) should be performed. Significant effort, however, should be directed toward stabilizing the process from the information in the routine samples.

**Figure 13.2**

*Mild Chaos with Low Fraction Defective*

If the fraction defective in the process under study wanders in an unpredictable manner so that the fractions for the best lots are above \( k_1/k_2 \), then 100 percent inspection should be performed, as shown in Figure 13.3. Again, significant effort should be directed toward stabilizing the process by using the information from 100 percent inspection.
These two cases are considered mild chaos because the distribution of the fraction defective is chaotic within bounds. Of course, these bounds can disappear at any moment. These are not situations where we should be lulled into thinking that the chaos will always stay within bounds. Chaos is like a wild beast that can run anywhere at anytime, including beyond any earlier boundary.

13.6.2 Severe Chaos

If the fraction defective in the process under study wanders in an unpredictable manner within a narrow range around \( k_1/k_2 \), the most practical plan is 100 percent inspection of all lots. No effort at acceptance sampling can justify the cost of administering the plan [Deming, 1950, p. 271].

If the fraction defective in the process under study wanders in an unpredictable manner within a wide range around \( k_1/k_2 \), Orsini has devised a rule that yields substantial savings over 100 percent inspection [Orsini, 1982]:

- If \( k_1/k_2 \) is less than 1/1,000, then inspect 100 percent of the incoming lots.
- If \( k_1/k_2 \) is between 1/1,000 and 1/100, then test a sample of 200. If there are no defectives, then accept the remainder. Inspect the entire remainder if at least one defective item is found in the sample.
- If \( k_1/k_2 \) is greater than 1/100, then do no inspection.
This rule is also helpful in working with a vendor to bring his process into control. A running record of the samples of 200 can be kept, and the number of defectives can be charted, sample by sample. Feedback to the vendor is extremely helpful in identifying problems.

13.6.3 Exceptions to Rules for Chaos

The rules for unstable processes are subject to the same exceptions as the kp rule for stable processes. Further, if items come into a firm from an unknown vendor, the optimal policy is to perform 100 percent inspection until enough information has been collected to construct a control chart(s) for the vendor’s process/product, we can then select the best inspection plan. Since for safety items \( k_2 \) is infinite, one hundred percent inspection should also be carried out for critical parts and safety items.

13.7 Summary

This chapter has discussed different types of acceptance sampling plans: lot-by-lot plans, continuous sampling plans, and special sampling plans. Lot-by-lot acceptance sampling plans are used to inspect goods or services whenever the goods or services can be conveniently grouped into lots. Continuous flow acceptance sampling plans are used to inspect goods or services whenever the goods or services cannot be grouped into lots -- for example, goods on a conveyor belt. Special sampling plans are used for lots found in series. These acceptance sampling plans do not, however, minimize the average total cost of inspection of incoming, intermediate, and final goods and services. A theoretical argument invalidating acceptance sampling has been presented.

Plans for minimizing the average total cost of testing incoming materials and final product, called the kp rule, were analyzed. One plan is used for stable processes, and the other plan is used for chaotic processes, subject to exceptions when testing is destructive or we are dealing with homogeneous mixtures. Both plans require the collection and control charting of either inspection data or routine samples to achieve process stability and pursue continuous and never-ending improvement.

EXERCISES

13.1 Discuss the three possible alternatives for the inspection of goods or services.

13.2 (a) Explain the purpose of acceptance sampling.
(b) Explain the purpose of lot-by-lot acceptance sampling plans. Describe the situations in which lot-by-lot acceptance sampling plans are used as a basis for action on a lot of goods.
(c) Explain the purpose of Military Standard 414.
(d) Explain the purpose of Military Standard 105D.
(e) Explain the purpose of continuous flow acceptance sampling plans.
(f) Briefly describe the operation of continuous flow acceptance sampling plans.
13.3 Explain why acceptance sampling plans are theoretically incorrect for stable processes and should not be used as a basis for action. Mathematically defend your explanation.

13.4 Explain why acceptance sampling is theoretically correct but not economical for chaotic processes and consequently should not be used as a basis for action.

13.5 (a) Describe the kp rule for stable processes.
(b) Explain the assumptions required to use the kp rule.
(c) List several examples of $k_1$ inspection costs.
(d) List several examples of $k_2$ inspection costs.

13.6 Explain the term mild chaos and its significance to taking action on incoming or intermediary material or on final product.

13.7 A radio manufacturer has a policy of inspecting every incoming radio speaker to ensure it conforms to specifications. What information would you need to question the wisdom of this inspection policy?

13.8 The production manager of Exercise 13.7's radio company learned about the kp rule. He used past inspection data concerning the proportion of defective radio speakers purchased per day to construct a p chart with variable sample size. The p chart indicated that the incoming stream of radio speakers was stable with respect to the fraction of defective radio speakers purchased each day. The average fraction of defective radio speakers was found to be 0.002. Further study showed that it costs approximately $0.50 to inspect an incoming radio speaker and that it costs approximately $7.50 to repair a radio with a defective speaker before it leaves the factory.
(a) Use the kp rule to determine if 0 percent of 100 percent inspection should be used for incoming radio speakers.
(b) What should management do about the incoming radio speaker process, given your answer in part (a)?

13.9 Explain the term severe chaos. Describe an inspection rule that can be used when a process exhibits severe chaos.

REFERENCES AND ADDITIONAL READINGS


APPENDIX 13.1

Proof That the Number of Defectives in a Sample is Independent of the Number of Defectives in the Remainder for Lots Drawn from a Sable Process

To prove: Given a stable process with 100p percent defective, the number of defectives in a sample of n items drawn from a lot of size N is independent of the number of defectives in the remaining N - n items.

Proof:  

\[ n = \text{number of items in the sample} \]
\[ N = \text{number of items in the lot} \]
\[ r = \text{number of defective items in the sample} \]
\[ x = \text{number of defective items in the lot} \]
\[ x - r = \text{number of defective items in remaining N - n items} \]
\[ f(x) = \text{the probability of x defectives in a lot of N items drawn from a stable process with 100p percent defective} \]
\[ f(r) = \text{the probability of r defectives in a sample of n items drawn from a stable process with 100p percent defective} \]
\[ f(r | x) = \text{the conditional probability of r defectives in a sample of n items drawn from a lot of N items given that the lot contains x defectives} \]
\[ f(r \text{ and } (x - r)) = \text{the joint probability that there are r defectives in the sample of n items and x - r defectives in the remaining N - n items} \]

We must show that \( f(r \text{ and } (x - r)) = f(r)f(x - r) \).

The joint probability of x defectives in the lot and r defectives in the sample is given by

\[ f(x \text{ and } r) = f(x)f(r | x) \]

The number of defectives, x, in a lot of size N is binomially distributed:

\[ f(x) = \binom{x}{N} p^x q^{N-x} \]

Similarly, the number of defectives, r, in a sample of size n is binomially distributed:

\[ f(r) = \binom{n}{r} p^r q^{n-r} \]

and the number of defectives, x - r, in the remaining N - n items is also binomially distributed:

\[ f(x - r) = \binom{N-n}{x-r} p^{x-r} q^{N-n} - (x-r) \]

The number of defectives, r, in a sample of size n, given a total of x defectives in a lot of size N, has a hypergeometric distribution, given by
\[ f(r \mid x) = \binom{n}{r} \frac{\binom{N-n}{x-r}}{\binom{N}{x}} \]

Then

\[ f(x \text{ and } r) = \binom{n}{r}^x p^x q^{N-x} \frac{\binom{n}{r} \binom{N-n}{x-r}}{\binom{N}{x}} \]

This can be written as:

\[ f(x \text{ and } r) = p^{x+r} q^{N-x-n+r} \binom{n}{r} \binom{N-n}{x-r} \]

or, rearranging

\[ f(x \text{ and } r) = \binom{N-n}{x-r} p^{x-r} q^{(N-n)-(x-r)} \binom{n}{r} r^{n-r} \]

or

\[ f(x \text{ and } r) = f(x - r)f(r) \]

This is the probability of \( x \) defectives in a lot of size \( N \) and \( r \) defectives in a sample of size \( n \). But this is the same as the probability of \( x - r \) defectives in \( N - n \) items and \( r \) defectives in \( n \) items. So

\[ f(x \text{ and } r) = f((x - r) \text{ and } r) \]

Then

\[ f((x - r) \text{ and } r) = f(r)f(x - r) \]

If the joint probability of two events equals the product of their unconditional probabilities, the events must be independent. Thus, the number of defectives in a sample of \( n \) items is independent of the number of defectives in the remaining \( N - n \) items.
APPENDIX 13.2
Derivation of the kp Rule for Stable Processes

Let

- \( p \) = the average fraction of defective items in incoming lots of items
- \( k_1 \) = the cost to initially inspect one item
- \( k_2 \) = the cost to dismantle, repair, reassemble, and test a good or service that fails because a defective item was used in its production
- \( k \) = the average cost to test one or more items to find a conforming item from the supply, \( S \), to replace a defective item found
  \[ k = \frac{k_1}{1 - p} \]
- \( C_1 \) = the cost to initially inspect one item
- \( C_2 \) = the cost to repair a failed good or service

Further, let

\[ x_i = \begin{cases} 1 & \text{if item i is defective, and} \\ 0 & \text{if item i is conforming} \end{cases} \]

Now, if one item, item i, is randomly drawn from a lot, then the probability that it is defective is \( p \). The cost to initially inspect item i is

\[ C_1 = \begin{cases} k_1 + kx_i & \text{if we test item i, and} \\ 0 & \text{if we do not test item i} \end{cases} \]

\( C_1 \) is composed of the cost to initially test one item plus the cost to replace the item if it is found to be defective. Hence,

\[ C_1 = \begin{cases} k_1 + k & \text{if item i is tested and found to be defective,} \\ k_1 & \text{if item i is tested and found to conform, or} \\ 0 & \text{if item i is not tested.} \end{cases} \]

The cost to repair a failed good or service due to item i is

\[ C_2 = \begin{cases} (k_2 + k)x_i & \text{if we do not initially test item i, and} \\ 0 & \text{if we do initially test item i.} \end{cases} \]

\( C_2 \) is composed of the cost to repair a failed good or service if item i was not initially inspected. Hence

\[ C_2 = \begin{cases} k_2 + k & \text{if item i was not initially inspected and item i is defective,} \\ 0 & \text{if item i was not initially inspected and item i is conforming, and} \\ 0 & \text{if item i was initially inspected.} \end{cases} \]
$C_1$ and $C_2$ are mutually exclusive as they cannot occur simultaneously for a given item; if one is positive, the other is zero. The total cost for item $i$ is

$$C = C_1 + C_2$$

Table A13.1 summarizes the cost structure for inspection versus no inspection for item $i$ [Deming, 1950, p. 302].

<table>
<thead>
<tr>
<th>Inspect the Item?</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Total Cost $C = C_1 + C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>$k_1 + kx_i$</td>
<td>0</td>
<td>$k_1 + kx_i$</td>
</tr>
<tr>
<td>No</td>
<td>0</td>
<td>$(k_2 + k)x_i$</td>
<td>$(k_2 + k)x$</td>
</tr>
</tbody>
</table>

Table A13.2 extends the cost structure for inspection versus no inspection to the average cost per item over the lot. In this case $x_i$ is replaced by $p$ because

$$p = \sum_{i=1}^{N} \left[ \frac{x_i}{N} \right]$$

None of the other elements in Table A13.2 is affected because they are constants.

<table>
<thead>
<tr>
<th>Inspect The Item?</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Total Cost $C = C_1 + C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>$k_1 + kp$</td>
<td>0</td>
<td>$k_1 + kp$</td>
</tr>
<tr>
<td>No</td>
<td>0</td>
<td>$(k_2 + k)p$</td>
<td>$k_2p + kp$</td>
</tr>
</tbody>
</table>

Now, the breakeven point between inspection and no inspection can be determined by setting the total cost for inspection equal to the total cost for no inspection. That is, at the breakeven point, Cost (Inspect) = Cost (Do Not Inspect):

$$k_1 + kp = k_2p + kp$$

$$k_1 = k_2p$$

$$p = k_1/k_2$$