CHAPTER 11

PROCESS CAPABILITY AND IMPROVEMENT STUDIES

Sections

Introduction
Specifications (Voice of the Customer) and Created Dimensions
Process Capability Studies
Process Improvement Studies
Quality Improvement Stories
Summary
Exercises
References

Chapter Objectives

• To discuss the customer specification limits (Voice of the Customer) for the output of a process (Voice of the Process)
• To define and illustrate performance specifications
• To define and illustrate technical specifications, including individual unit specifications, acceptable quality level (AQL) specifications and distribution specifications
• To discuss the distinction between performance and technical specifications
• To discuss the fallacy that conformance to technical specifications is sufficient to achieve acceptable quality
• To discuss the Voice of the Process, or the output of a process
• To discuss the importance and application of process capability studies to compare the Voice of the Process, or process performance, with the Voice of the Customer, or customer needs
• To describe and illustrate attribute process capability studies
• To describe and illustrate variables capability studies, and compare natural limits on the output of a stable process with specification limits
• To define and illustrate process capability indices to summarize processes
• To describe and illustrate two types of process improvement studies: attribute improvement studies and variables improvement studies
• To discuss and illustrate the use of quality improvement stories to present process capability and process improvement studies to management
11.1 Introduction

Process capability studies determine whether a process is unstable, investigate any sources of instability, determine their causes, and take action to resolve such sources of instability. After all sources of instability have been resolved in a process, the natural behavior of the process is called its process capability. Process capability compares the output of a process (called "Voice of the Process") with the customer's specification limits for the outputs (called "Voice of the Customer"). A process must be stable (or have an established process capability) before it can be improved. Consequently, a process capability study must be successfully completed before a process improvement study can have any chance for success.

Process improvement studies follow the Deming cycle of Plan, Do, Study, Act. First, managers construct a plan to decrease the difference between customer needs (Voice of the Customer) and process performance (Voice of the Process). Recall that a plan is an intention to move from an existing method or flowchart to a revised method or flowchart by incorporating one or more change concepts. Second, they test the revised flowchart's (Plan) viability using a planned experiment (Do). Third, they collect data and study the results of the planned experiment to determine if the plan (revised flowchart) will decrease the difference between customer needs and process performance (Study). Fourth, if the data collected about the revised flowchart show that the plan will achieve its objective(s), the revised flowchart is standardized through "best practices" and training (Act); the managers responsible for the plan return to the Plan phase of the Deming cycle to find further revisions to the flowchart that will continue to reduce the difference between customer needs and process performance. If the data collected about the plan show that the plan will not achieve its objective(s), the managers responsible for the plan return to the Plan phase of the Deming cycle to find a different revision to the flowchart that will reduce the difference between customer needs and process performance. Hence, the Deming cycle follows a never-ending path of process and quality improvement.

This chapter is divided into four sections: specifications, process capability studies, process improvement studies, and quality improvement stories. The quality improvement story is an effective format for quality management practitioners to present process capability and process improvement studies to management.

11.2 Specifications (Voice of the Customer) and Created Dimensions

Specifications fall into two broad categories: performance specifications and technical specifications.

11.2.1 Performance Specifications

Performance specifications address a customer's needs or wants. An example of a performance specification can be seen in restaurants rated by the Red Michelin Guide. The customers of these restaurants set their performance specifications as “a perfect dining experience.” Perfection is measured in terms of the synergistic experience created
by the interaction of food, service, ambience and price. The *Red Michelin Guide* rates restaurants on a one to three star scale. Only the best restaurants in the world receive Michelin stars. A restaurant receives one Michelin star for consistently serving very good food in a good setting, but it is not considered worthy of a special traveling effort. A restaurant receives two Michelin stars for consistently serving excellent food, including specialties and wines of choice in a great setting. The restaurant is worth a detour from one’s existing travel itinerary. A restaurant receives three Michelin stars for serving excellent food and great wine, with impeccable and elegant service and ambience. The restaurant is one of the best restaurants in the world and is worth a special trip. All starred restaurants have a high average level of quality with very little variation around the average. A three star Michelin chef is an artist; it is as if Picasso were painting for your pleasure. Three star Michelin restaurants provide performance specifications. They guarantee satisfaction at the point of delivery. Nothing short of perfection is acceptable.

11.2.2 Technical Specifications

**Technical specifications** describe the desired values of quality characteristics at delivery. There are three types of technical specifications: **individual unit specifications; acceptable quality level (AQL) specifications; and distribution specifications.**

**Individual Unit Specifications.** Individual unit specifications state a boundary (upper or lower specification limit), or boundaries (both upper and lower specification limits), that apply to individual units of a product or service. An individual unit of product or service is considered to conform to a specification if it is on or inside the boundary or boundaries; this is the **goal post view of quality.** Individual unit specifications are made up of two parts, which together form a third part. The first part of an individual unit specification is the **nominal, or target value.** This is the desired value for process performance mandated by the customer's needs. Ideally, if all quality characteristics were at nominal, products and services would perform as expected over their life cycle. The second part of an individual unit specification is a **tolerance.** A tolerance is an allowable departure from a nominal value established by design engineers that is deemed non-harmful to the functioning of the product or service over its life cycle. Tolerances are added and/or subtracted from nominal values. The third part of an individual unit specification is a **specification limit,** or the boundaries created by adding and/or subtracting tolerances from a nominal value. It is possible to have two-sided specification limits:

\[
\text{USL} = \text{Nominal} + \text{Tolerance} \\
\text{LSL} = \text{Nominal} - \text{Tolerance}
\]

where USL is the upper specification limit and LSL is the lower specification limit; or one-sided specification limits (i.e., either USL or LSL only). A nominal value together with specification limits form the Voice of the Customer.

An example of an individual unit specification and its three parts can be seen in the specification for the "case hardness depth" of a camshaft. A camshaft is considered to be
conforming with respect to case hardness depth if each individual unit is between 7.0 mm ± 3.5 mm (or LSL = 3.5 to USL = 10.5 mm). The nominal value in that specification is 7.0 mm; the two-sided tolerance is 3.5 mm; the lower specification limit is 3.5 mm (7.0 mm - 3.5 mm); and the upper specification limit is 10.5 mm (7.0 mm + 3.5 mm).

From our earlier discussion of the philosophy of continuous reduction of variation (i.e., the Taguchi Loss Function), we saw that the goal of modern management should not be 100 percent conformance to specifications (Zero Defects), but the never-ending reduction of process variation within specification limits so that all products/services are as close to nominal as possible, absent capital investment. Specified tolerances become increasingly irrelevant as process variation is reduced so that the process's output is well within specification limits.

**Acceptable Quality Level (AQL) Specifications.** Acceptable quality level (AQL) specifications state a requirement that must be met by most individual units of product or service, but allow a certain proportion of the units to exceed the requirements. For example, camshafts shall be acceptable if no more than 3 percent of the units exceed the specification limits of 3.5 and 10.5 mm. This type of specification limit is frequently referred to as an Acceptable Quality Level. AQL specifications are much like individual unit specifications, except they have a unique negative feature: they formally support the production of a certain percentage of defective product or service.

**Distribution Specifications.** Distribution specifications define an acceptable distribution for each product or service quality characteristic. In an analytic study, a distribution is defined in terms of its mean, standard deviation, and shape. However, from the Empirical Rule discussed in Chapter 5, it is not necessary to make rigid assumptions about the shape of the distribution. That is, virtually all data from a stable process will fall between the mean plus or minus three standard deviations.

As an example of a distribution specification, the case hardness depth of a camshaft shall be stable with an average depth of 7.0 mm and a standard deviation not to exceed 1.167 mm. In other words, individual units shall be distributed around the average with a dispersion not to exceed 3.50 mm on either side of the average since for a stable process, virtually all of the output will be within three standard deviations on either side of the mean [7.0 mm ± 3(1.167 mm) = 7.0 mm ± 3.50 mm = 3.50 to 10.50 mm]. The mean and standard deviation are simply directional goals for management when using distribution specifications. Management must use statistical methods to move the process average toward the nominal value of 7.0 mm and to decrease the process standard deviation as far below 1.167 mm as possible. Distribution requirements are stated in the language of the process and promote the never-ending improvement of a process.

**Distinguishing between Performance Specifications and Technical Specifications.** Performance specifications are not commonly used in business; instead, technical specifications are used. Unfortunately, this can cause major problems because technical specifications may not produce the performance desired by a customer. As an example, consider a hospital that serves medium (versus rare or well-done) steak to patients who
select steak for dinner. [Camp, 1986] The performance desired is patient satisfaction within nutritional guidelines. But performance specifications are not used. Instead, a technical specification of five ounces of steak is substituted; it is assumed they are equivalent.

A hospital purchasing agent switches from meat vendor A to meat vendor B to secure a lower price, while still meeting the technical specification of five ounces. He does not discuss or inform the hospital nutritionist and kitchen staff of the switch in vendors. The hospital nutritionist begins receiving complaints from patients that the steak is tough and well-done. She investigates and finds that vendor A's steaks were thick, while vendor B's are thin (but longer and wider). She realizes, using an I-MR chart of 60 successive steak temperatures that the thinner steaks get hotter more quickly, and hence, cook faster, given the usual preparation regimen, as shown in Figure 11.1. She concludes, "If I'd known that the steaks had been changed, I could have accommodated the change without creating patient dissatisfaction." The purchasing agent says, "I met the technical specification of five ounces." The problem lies in assuming that technical specifications are the same as desired level of performance. This is not necessarily true.

![I-MR Chart of Temperature by Supplier](image)

**Figure 11.1**
I-MR chart of Cooked Temperature of 5-Ounce Hospital Steaks

*The Fallacy That Conformance to Technical Specifications Defines Quality.* Mere conformance to specification limits is insufficient to achieve the quality level required to compete effectively in today's marketplace. Management must constantly try to reduce process variation around a nominal value, within specification limits (i.e., the Taguchi Loss Function) to achieve the degree of uniformity required to produce products or services that function exactly as promised to the customer over their life cycle.
**Created Dimensions.** The features of products, services or processes that are created when the component parts of products or services are assembled are called created dimensions. The Voice of the Customer for created dimensions is discussed below.

When parts are assembled, new dimensions are created; these new dimensions have statistical distributions. [AT&T, 1956, pp. 119-127] For example, if two boards are glued together to form a double-thick board, the distribution of the thickness of the double-thick boards is a newly-created dimension. Management must be able to control and reduce the variation of these created dimensions so the final assemblies will perform as the customer desires over the product's life cycle. Understanding and controlling these created dimensions requires working knowledge of the statistical rules of created dimensions. If management does not pay attention to their statistical characteristics, these dimensions will fail to be within specification limits and will cause problems in production or service, will increase costs, and will lead to customer dissatisfaction. The discussion of specifications earlier in this chapter applies to created dimensions as well.

**Law of the Addition of Component Dimension Averages.** If component parts are assembled so that the individual component dimensions are added to one another, the average dimension of the assembly will equal the sum of the individual component average dimensions. Figure 11.2 illustrates this concept. If three component parts are glued together (assuming the glue takes no measurable dimensions), the average width of the assembled part equals the sum of the average individual part widths, as shown in Equation 11.1:

$$X_{\text{assembly}} = X_1 + X_2 + X_3$$

(11.1)

where

- $X_{\text{assembly}}$ = Average width of the assembly
- $X_1$ = Average width of part 1,
- $X_2$ = Average width of part 2, and
- $X_3$ = Average width of part 3.
Part 1 has an average thickness of 10 mm, part 2 has an average thickness of 20 mm, and part 3 has an average thickness of 30 mm. Consequently, the average thickness of the final assembly is the sum of all three averages, 60 mm (10 mm + 20 mm + 30 mm):

\[ \overline{X}_{\text{assembly}} = \overline{X}_1 + \overline{X}_2 + \overline{X}_3 \]
\[ = 10 \text{ mm} + 20 \text{ mm} + 30 \text{ mm} \]
\[ = 60 \text{ mm} \]

The preceding law holds only if the processes generating the components are in statistical control.

**Law of the Differences of Component Dimension Averages.** If component parts are assembled so that the individual component dimensions are subtracted from one another, the average dimension of the assembly will then equal the difference between the individual component average dimensions. Figure 11.3 illustrates this concept. If a bolt is projected through a steel plate, the average length of the bolt projection through the steel plate equals the difference between the bolt's shank length and the width of the steel plate, as shown in Equation 11.2:
\[ \bar{X}_{\text{bolt projection}} = \bar{X}_s - \bar{X}_p \]  \hfill (11.2)

where

- \( \bar{X}_{\text{bolt projection}} \) = Average length of the bolt shank projection through the steel plate,
- \( \bar{X}_s \) = Average length of the bolt shank, and
- \( \bar{X}_p \) = Average width of the steel plate.

**Figure 11.3**

**Differences of Averages**

The bolt shank has an average length of 12 mm, and the steel plate has an average width of 8 mm. Consequently, the average bolt projection through the steel plate is 4 mm (12 mm - 8 mm):

\[ \bar{X}_{\text{bolt projection}} = \bar{X}_s - \bar{X}_p = 12 \text{ mm} - 8 \text{ mm} = 4 \text{ mm} \]

Again, the preceding law holds only for component processes in statistical control.

**Law of the Sums and Differences of Component Dimension Averages.** If component parts are assembled so that the individual component parts are added and subtracted from one another, the average dimension of the assembly will then equal the algebraic sum of the individual component average dimensions. Figure 11.4 illustrates this concept. If a bolt is screwed through a steel plate and washers are inserted on either side of the plate, the average length of the bolt projection through the steel plate and washers then equals the difference between the sum of the widths of the two washers and the steel plate, and the length of the bolt shank, as shown in Equation 11.3:

\[ \bar{X}_{\text{bolt projection}} = \bar{X}_s - (\bar{X}_{w1} + \bar{X}_p + \bar{X}_{w2}) \]  \hfill (11.3)

where

- \( \bar{X}_{\text{bolt projection}} \) = Average length of the bolt shank projection through both washers and the steel plate,
- \( \bar{X}_s \) = Average length of the bolt shank,
- \( \bar{X}_{w1} \) = Average width of the top washer,
\( \bar{X}_{w2} = \) Average width of the bottom washer, and
\( \bar{X}_p = \) Average width of the steel plate.

**Figure 11.4**
Sums and Differences of Averages

The bolt shank has an average length of 40 mm, the steel plate has an average thickness of 27 mm, the top washer has an average thickness of 3 mm, and the bottom washer has an average thickness of 4 mm. Hence, the average bolt projection through the steel plate and both washers is 6 mm [40 mm - (3 mm + 27 mm + 4 mm)]:

\[
\bar{X}_{\text{bolt projection}} = \bar{X}_s - (\bar{X}_{w1} + \bar{X}_p + \bar{X}_{w2}) \\
= 40 \text{ mm} - (3 \text{ mm} + 27 \text{ mm} + 4 \text{ mm}) \\
= 6 \text{ mm}
\]

Again, the preceding law holds only for component processes in statistical control.

**Law of the Addition of Component Dimension Standard Deviations.** If component parts are assembled at random (for example, so that each component part is drawn randomly from its own bin with no selection criteria), the standard deviation of the assembly will be the square root of the sum of the component variances, regardless of whether the components are added or subtracted from each other. This law applies to assemblies in which the component parts combine linearly and are statistically independent.

For example, consider again the bolt projection in Figure 11.4. Recall that \( \bar{X}_s = 12 \text{ mm} \) and \( \bar{X}_p = 8 \text{ mm} \); consequently, we found from Equation 11.3 that \( \bar{X}_{\text{bolt projection}} = 4 \text{ mm} \). Further, assume that the standard deviation of the bolt shank length, \( \sigma_s \), is 0.010 mm and
the standard deviation of the steel plate width, \( \sigma_p \), is 0.008 mm. The standard deviation of the bolt projection is shown in Equation 11.4; it is:

\[
\sigma_{\text{bolt projection}} = \left[ \sigma_s^2 + \sigma_p^2 \right]^{1/2} = \left[ (0.010)^2 + (0.008)^2 \right]^{1/2} = 0.0128 \text{ mm}
\]

We must realize that the standard deviation of the bolt projection is not 0.018 mm, the sum of the individual component standard deviations. The square root of the sum of the individual component variances will always be less than the sum of the individual component standard deviations. This means that the assembly-to-assembly variation among random assemblies will be less than would be indicated by summing the individual components' unit-to-unit variations. Again, the preceding law holds only for component processes in statistical control.

**Law of the Average for Created Areas and Volumes.** If areas and volumes are created by the assembly of component parts, then the average area or volume of the assembly will equal the product of the individual component average dimensions if the component processes are stable and independent. Figure 11.5 illustrates this concept. If a boxlike container is constructed with two short sides, two long sides, and two top/bottom sides, then the average internal volume of the boxlike container equals the product of the average length of the short side, the average length of the long side, and the average width of the sides, as shown in Equation 11.5:

\[
\bar{X}_v = (\bar{X}_s)(\bar{X}_l)(\bar{X}_w)
\]

where

\[
\begin{align*}
\bar{X}_v & = \text{Average internal volume of the constructed container}, \\
\bar{X}_s & = \text{Average length of short side}, \\
\bar{X}_l & = \text{Average length of long side}, \text{ and} \\
\bar{X}_w & = \text{Average width of the sides}.
\end{align*}
\]

**Figure 11.5**

*Created Volume of a Container*

\[
\bar{x}_l = 8.0\text{ mm}, \sigma_s = 1.0\text{ mm} \\
\bar{x}_w = 2.0\text{ mm}, \sigma_w = 0.20\text{ mm} \\
\bar{x}_s = 3.0\text{ mm}, \sigma_s = 0.25\text{ mm}
\]
The average length of the short side is 3.0 mm, the average length of the long side is 8.0 mm, and the average width of the sides is 2.0 mm. Consequently, the average internal volume of the constructed container is 48 mm$^3$ (3 mm x 8 mm x 2 mm).

$$\bar{X}_v = (\bar{X}_s) (\bar{X}_l) (\bar{X}_w) = (3 \text{ mm})(8 \text{ mm})(2 \text{ mm}) = 48 \text{ mm}^3$$

Again, the preceding law holds only for component processes in statistical control.

**Law of the Standard Deviation for Created Areas and Volumes.** If areas and volumes are created by the assembly of component parts, we can calculate the standard deviation of the created areas, shown in Equation 11.6, or volumes, shown in Equation 11.7:

$$\sigma_{\text{area}} = \left[ \bar{X}_s^2 \sigma_s^2 + \bar{X}_l^2 \sigma_l^2 + \bar{X}_w^2 \sigma_w^2 \right]^{1/2} \quad (11.6)$$

where

- $\bar{X}_s =$ Average length of the short side,
- $\bar{X}_l =$ Average length of the long side,
- $\sigma_s =$ Standard deviation of length of the short side,
- $\sigma_l =$ Standard deviation of length of the long side, and
- $\sigma_{\text{area}} =$ Standard deviation of the created internal area.

Hence

$$\sigma_{\text{volume}} = \left[ \bar{X}_s^2 \bar{X}_w^2 \sigma_s^2 + \bar{X}_l^2 \bar{X}_w^2 \sigma_l^2 + \bar{X}_s^2 \bar{X}_l^2 \sigma_s^2 + \bar{X}_s^2 \sigma_l^2 \sigma_s^2 + \bar{X}_l^2 \sigma_s^2 \sigma_l^2 + \bar{X}_w^2 \sigma_s^2 \sigma_l^2 \right]^{1/2} \quad (11.7)$$

where

- $\bar{X}_w =$ Average width,
- $\sigma_w =$ Standard deviation of the width.

All equations assume that the component processes are stable and independent. Figure 11.5 illustrates this concept. If the means and standard deviations for the boxlike container’s dimensions are as shown in Figure 11.5, then the standard deviation of the internal volume for the assembled container is:

$$\sigma_{\text{volume}} = \left[ \bar{X}_s^2 \bar{X}_w^2 \sigma_s^2 + \bar{X}_l^2 \bar{X}_w^2 \sigma_l^2 + \bar{X}_s^2 \bar{X}_l^2 \sigma_s^2 + \bar{X}_s^2 \sigma_l^2 \sigma_s^2 + \bar{X}_l^2 \sigma_s^2 \sigma_l^2 + \bar{X}_w^2 \sigma_s^2 \sigma_l^2 \right]^{1/2}$$

$$= [3^2(2^2)(.05)^2 + 8^2(2^2)(.25)^2 + 3^2(8^2)(.20)^2 \right.$$  
$$+ 3^2(1^2)(.20)^2 + 8^2(25^2)(.20)^2 + 2^2(25^2)(1^2) \right.$$  
$$+ (1^2)(25^2)(.20)^2]^{1/2}$$

$$= [36 + 16 + 23.04 + .36 + .16 + .25 + .0025]^{1/2}$$

$$= (75.8125)^{1/2}$$

$$= 8.71 \text{ mm}^3$$

Again, the preceding law holds only for component processes in statistical control.
Process capabilities can be computed for created dimensions as follows. For example, suppose that the process capability studies for the thickness of the sheet, pin, and both washer subcomponents of the assembly in Figure 11.4 were based on subgroups of five subcomponents and yielded:

\[ \bar{x}_s = 40 \text{ mm and } \sigma_s = 0.0050 \]
\[ \bar{x}_{w1} = 3 \text{ mm and } \sigma_{w1} = 0.0007 \]
\[ \bar{x}_{w2} = 4 \text{ mm and } \sigma_{w2} = 0.0008 \]
\[ \bar{x}_p = 27 \text{ mm and } \sigma_p = 0.0030 \]

Further, assume that all component processes are independent of each other and stable. The average bolt projection is:

\[ \bar{x}_{\text{bolt projection}} = 40 \text{ mm} - 3 \text{ mm} - 27 \text{ mm} - 4 \text{ mm} = 6 \text{ mm} \]

The standard deviation of the bolt projection is:

\[ \sigma_{\text{bolt projection}} = \left[ (0.0050)^2 + (0.0007)^2 + (0.0008)^2 + (0.0030)^2 \right]^{1/2} \]
\[ = 0.00593 \approx 0.006 \]

The process capability of the created bolt projection dimension is computed using as follows:

\[ \text{UNL} = 6 \text{ mm} + 3(0.006) = 6 \text{ mm} + 0.018 \text{ mm} = 6.018 \text{ mm} \]
\[ \text{LNL} = 6 \text{ mm} - 3(0.006) = 6 \text{ mm} - 0.018 \text{ mm} = 5.982 \text{ mm} \]

From the normal distribution, 99.73% of the bolt projections will be between 5.982 mm and 6.018 mm. Alternatively, using the Empirical Rule, virtually all of the bolt projections will be between 5.982 mm and 6.018 mm.

Bolt projection specifications are set at 5.99 ± 0.02 mm, or USL = 6.01 mm and LSL = 5.97 mm. Consequently,

\[ Z_{\text{USL}} = \frac{(\text{USL} - \bar{x})}{\sigma} \]
\[ Z_{\text{USL}} = \frac{(6.01 - 6.00)}{0.006} = \frac{0.01}{0.006} = 1.67 \]

The process average is only 1.67 standard deviations below the upper specification limit, indicating that some output will be nonconforming. The fraction of output that was nonconforming in the period under study can be determined by examining the histogram of the output, as in our camshaft example.
\[
Z_{LSL} = \frac{(\bar{x} - LSL)}{\sigma} = \frac{(6.00 - 5.97)}{0.006} = \frac{0.03}{0.006} = 5.00
\]

The process average is 5.00 standard deviations above the lower specification limit, indicating the likely production of only conforming output.

Finally, created dimensions should be control charted because a created dimension can be out of control while component dimensions are in control. Never-ending improvement cannot progress without reducing unit-to-unit variations and moving the process toward nominal for created dimensions.

### 11.3 Process Capability Studies

There are two types of process capability studies: attribute process capability studies and variables process capability studies.

#### 11.3.1 Attribute Process Capability Studies

**Attribute process capability studies** determine a process's capability in terms of fraction defective output or counts of defects for a unit of output. The major tools used in attribute process capability studies are attribute control charts, discussed in Chapter 7, and the diagnostic tools discussed in Chapter 10. The process capability for a p chart is \( \bar{p} \), the average fraction defective units generated by the process. The process capability for the np chart is \( np \), the average number of defective units generated by the process for a given subgroup size, \( n \). The process capability for a c chart is \( \bar{c} \), the average number of defects per unit generated by the process for a given area of opportunity. Finally, the process capability for a u chart is \( \bar{u} \), the average number of defects per unit generated by the process where the area of opportunity varies from subgroup to subgroup.

A shortcoming of this type of study is that it begins with a specification, but it is not specific about the reason for failure to meet that specification. The p chart does not indicate if defective units result from the process being off nominal and too close to the specification limit, or because the process has too much unit-to-unit variation, or because the process is not stable with respect to its mean and/or variance. Further, as p charts are relatively insensitive to shifts or trends in the process, problems can go undetected for so long that they cause defectives before they are checked. p charts are frequently based on readily available data.
11.3.2 Variables Process Capability Studies

**Variables process capability studies** determine a process's ability to meet specifications stated by the customer. The major tools used in variables process capability studies are variables control charts, discussed in Chapter 8, and the diagnostic tools discussed in Chapter 10. Variables control charts are used to stabilize a process so we can determine meaningful upper and lower natural limits. **Natural limits** are computed for stable processes by adding and subtracting three times the process's standard deviation to the process centerline. In general, for any variables control chart, the upper and lower natural limits are:

\[
UNL = \bar{x} + 3\sigma \\
LNL = \bar{x} - 3\sigma
\]  

(11.8)  

(11.9)

For \( \bar{x} \) and R charts specifically, the upper and lower natural limits are:

\[
UNL = \bar{x} + 3(\bar{R}/d_2) \\
LNL = \bar{x} - 3(\bar{R}/d_2)
\]  

(11.10)  

(11.11)

where \( d_2 \) is a constant factor based on subgroup size that is presented in Table B.1 in Appendix B. \( d_2 = (\bar{R} / \text{sigma}) \) for a stable normal distribution.

For \( \bar{x} \) and s charts, the upper and lower natural limits are:

\[
UNL = \bar{x} + 3(s/c_4) \\
LNL = \bar{x} - 3(s/c_4)
\]  

(11.12)  

(11.13)

where \( c_4 \) is a constant factor based on subgroup size that is presented in Table B.1 in Appendix B. \( c_4 = (s / \text{sigma}) \) for a stable normal distribution.

For individuals charts, the upper and lower natural limits are:

\[
UNL = \bar{x} + 3(\bar{R}/d_2) \\
LNL = \bar{x} - 3(\bar{R}/d_2)
\]  

(11.14)  

(11.15)

where, \( d_2 \) is a constant factor based on a subgroup size of 2 that is shown in Table B.1 in Appendix B. \( d_2 = (\bar{R} / \text{sigma}) \) for a stable normal distribution.

As a rule, natural limits should not be shown on variables control charts because natural limits apply to individual units of output and control limits apply to subgroup statistics. One
notable exception to this rule is the individuals control chart for variables. In that case, the subgroups consist of individual units, and natural limits and control limits are the same.

Interpretation of the natural limits requires stability of the process under study and the application of the Empirical Rule discussed in Chapter 5. If the output distribution of a process is stable, then for Equations 11.8 through 11.15 we can say that virtually all process output will be between the natural limits. For example, if samples of four steel ingots are drawn from an ingot-producing process every hour, and the process is stable with a process average subgroup weight of 42.0 pounds (\( \bar{x} = 42.0 \) pounds) and an average range of 0.6856 pounds, we can say the following about the process using Equations 11.10 and 11.11:

1. The process's upper natural limit is
   \[
   UNL = \frac{\bar{x}}{3} + 3\left(\frac{\bar{R}}{d_2}\right) = 42.0 + 3\left(\frac{0.6856}{2.059}\right) \\
   \approx 42.0 + 3(0.333) = 42.999 \approx 43.0 \text{ pounds}
   \]

2. The process's lower natural limit is
   \[
   LNL = \frac{\bar{x}}{3} - 3\left(\frac{\bar{R}}{d_2}\right) = 42.0 - 3\left(\frac{0.6856}{2.059}\right) \\
   \approx 42.0 - 3(0.333) = 41.001 \approx 41.0 \text{ pounds}
   \]

3. Virtually all (99.73% for a stable normal distribution) of the steel ingots produced will weigh between 41.0 and 43.0 pounds. This is what the steel ingot process is capable of producing; it is the identity of the process.

The disadvantage of variables process capability studies is that they frequently require the collection of special data. The advantages of variables process capability studies are that they provide information such as whether the process is centered on nominal, or exhibiting too much unit-to-unit variation, or unstable with respect to its mean and/or variation. Furthermore, these studies are sensitive to shifts in the process and are helpful in detecting trends or shifts in the process before they cause trouble. Finally, variables process capability studies examine if the specification limits are reasonable.

11.3.3 Data Requirements for Process Capability Studies

**Attribute Studies.** Attribute process capability studies require a great deal of data. In general, the study should cover at least three distinct time periods, where each time period should contain 20 to 25 samples and each sample should have between 50 and 100 units. This rule of thumb is based on empirical experience, as well as statistical theory.

**Variables Studies.** Variables process capability studies require far less data than attribute studies. However, a separate variables study may be required for each quality characteristic that can cause a unit to be defective. As a rule of thumb, a variables study should cover at least three distinct time periods. The first period should contain about 50
samples of between three and five units each, and the second and third time periods should contain 25 samples of between three and five units each.

**Addition of New Data onto a Process Capability Chart.** After initial control limits have been calculated, the question arises as to what to do with additional data: should revised control limits be computed, or should the old control limits be extended across the control chart and new points plotted against the old limits? Recall from our discussion in Chapter 8 on revising control limits that if the process is stable and has not changed significantly, new limits should not be calculated because they can stimulate tampering with the process. [AT&T, 1956, pp. 34-37 and 45–73] In this case, the best procedure is to plot the new data against the old limits and search for a change in the data pattern. If the process has changed significantly, new limits should then be calculated using only the data from the revised process. These new limits allow for analysis of the process’s new capability.

11.3.4 Process Capability Studies on Unstable Processes

Process statistics, such as the measures of location, dispersion, and shape discussed in Chapter 5, cannot be estimated from a process capability study performed on an unstable (chaotic) process; nevertheless, useful information is still available. In such cases, the study often reveals information about the sources of special variation that affect the process, and it provides an opportunity to better understand the process. [AT&T, 1956, pp. 34–37 and 45–73]

11.3.5 Process Capability Studies on Stable Processes

A process capability study on a stable process sets the stage for the estimation of the process’s central tendency, \( \bar{x} \), and standard deviation, \( \sigma \). These statistics allow: (1) comparisons between the process’s performance (Voice of the Process) and specifications (Voice of the Customer), and (2) the use of centerlines, or process averages, on which to establish budgets and forecasts. [Gitlow and Gitlow, 1987, p. 161] Note that predicting a stable process’s behavior in the near future assumes that the process will remain stable. Unfortunately, it is impossible to know if this will be the case, so caution is advised.

11.3.6 An Example of an Attribute Process Capability Study

The centerline on a stable attribute control chart should be used as an estimate of the overall process capability. But there is one important proviso: an estimate of overall process capability is not specific as to the potential cause or causes of defective output. To identify these, we must separate out all possible sources of defects (such as operators, machines, and vendors) and perform individual process capability studies for each source. In such a case, \( \bar{x} \) and R charts are often more cost-effective, in terms of sample size and information, than attribute charts, if they can be used.

To illustrate an attribute capability study, consider the case of a manager of a data entry department who has taken a survey indicating customer dissatisfaction. The manager
wants to determine the capability of the data entry operation in her department in terms of the proportion of defective entries produced. [Gitlow and Hertz, 1983, pp. 131-41] She decides to take samples of the first 200 lines of code from each day's output, inspect them for defects, and construct an initial p chart. Table 11.1 shows the raw data, and Figure 11.6 shows the initial p-chart. The latter reveals that on days 8 (14 defective lines out of 200 inspected) and 22 (15 defective lines out of 200 inspected) something special happened, not attributable to the system, to cause defective lines to be entered.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of lines inspected</th>
<th>Number of defective lines</th>
<th>Fraction of defective lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>5</td>
<td>.025</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>14</td>
<td>.070</td>
</tr>
<tr>
<td>9</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>11</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
<td>8</td>
<td>.040</td>
</tr>
<tr>
<td>13</td>
<td>200</td>
<td>2</td>
<td>.010</td>
</tr>
<tr>
<td>14</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>15</td>
<td>200</td>
<td>7</td>
<td>.035</td>
</tr>
<tr>
<td>16</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>17</td>
<td>200</td>
<td>3</td>
<td>.015</td>
</tr>
<tr>
<td>18</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>19</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>21</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>22</td>
<td>200</td>
<td>15</td>
<td>.075</td>
</tr>
<tr>
<td>23</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>24</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>4,800</strong></td>
<td><strong>102</strong></td>
<td>****</td>
</tr>
</tbody>
</table>

Table 11.1
Attribute Process Capability Study on Data Entry Operation
Raw Data for Construction of Control Chart
The manager calls a meeting of the 10 operators to brainstorm for possible special causes of variation on days 8 and 22. Results of the brainstorming session are put onto the cause-and-effect diagram in Figure 11.7. The 10 group members vote that their best guess for the problem on day 8 was a new untrained operator (see cause in Figure 11.7, circled in a cloud) who had been added to the work force, and that the one day it took the worker to acclimate to the new environment probably caused the unusually high number of errors. To ensure that this special cause will not be repeated, the manager institutes a one-day training program for all new employees. The 10 group members also vote that their best guess for the problem on day 22 was that on the previous evening the department had run out of paper from the regular vendor, did not expect a new shipment until the morning of day 23, and consequently purchased a one-day supply of paper from a new vendor. The operators found this paper was of inferior quality, which caused the large number of defective entries. To correct this special cause of variation, the manager revises the firm's relationship with its regular paper vendor and operationally defines acceptable quality for paper.
After eliminating the days for which special causes of variation are found, the manager recomputes the control chart statistics using Equations 6.1, 6.3, and 6.4:

Centerline($p$) = Average fraction of defective lines = $\bar{p}$

$$\bar{p} = \frac{73}{4,400} = 0.01659 \approx 0.017$$

$UCL(p) = 0.044$

$LCL(p) = 0.000$

Figure 11.8 shows the revised control chart. The process appears stable. The centerline and control limits were extended out into the future for 25 days. Data from daily samples...
of 200 lines of code were collected for these 25 days and plotted with respect to the forecasted centerline and control limits. The process was found to be stable. The capability of the process is such that it will produce an average of 1.7 percent defective lines per day. Further, the percentage defective will rarely surpass 4.4 percent. Although the process’s capability is now known, the manager is not satisfied with its capability and should not stop attempting further improvement. We will see how this is done in this chapter's section on process improvement studies.

Figure 11.8
Revised p Chart Following Removal of Special Causes

11.3.7 An Example of a Variables Process Capability Study

To illustrate a variables process capability study, consider an auto manufacturer who wishes to purchase camshafts from a vendor. [Ford Motor Company, 1983, pp. 7.E.9-7.E.18] The buyer is concerned with the finish grind, diameters, and case hardness as well as other quality characteristics of the camshaft. For illustrative purposes, this discussion will focus only on the case hardness depth of the camshafts. The contract between the auto manufacturer and camshaft vendor calls for camshafts that have an average case hardness depth of 7.0 mm (nominal) and are distributed around the average with a dispersion not to exceed 3.5 mm either way (tolerance); this is a distribution specification (Voice of the Customer). Further, the contract requires that the vendor produce a process capability study demonstrating statistical control of his process. Consequently, management's objective is to reduce camshaft-to-camshaft variation for case hardness.
depth and to move the process's average case hardness depth to the desired nominal of 7.0 mm (Voice of the Process).

A camshaft is a rod with elliptical lobes along its length. As the rod rotates, so do the elliptical lobes, and this ultimately causes intake and exhaust valves to open and close. The intake valves permit a mixture of fuel and air to enter the cylinders, where combustion takes place. The exhaust valves permit the waste gases to exit the cylinders after combustion. The surfaces of the elliptical lobes must be hardened (made brittle) to reduce wear, as shown in Figure 11.9. This hardening is called case hardening and is accomplished by immersing the camshaft in oil, placing electric bearing coils around the lobes, and passing electric current through the coils. This process heat treats the lobes and makes them brittle. The depth to which the brittleness extends is called the case hardness depth. The case hardness depth must be tightly controlled since if the case hardness depth is too deep, the lobes will be too brittle and will tend to crack, while if the case hardness depth is too shallow, the lobes will be too soft and will wear quickly.

Figure 11.9
Camshaft in an Engine
Pursuant to the terms of the contract calling for a process capability study, a sample of five camshafts is drawn from the vendor's process every day. Each shaft is measured with respect to each of the relevant quality characteristics (CAMSHAFT). Figure 11.10 shows the control chart for the initial data collected in the process capability study.

Figure 11.10
Process Capability Study of the Camshaft
These data reveal that the vendor's process is not in statistical control; this is indicated by points 3, 12, 16, and 30 in the R-chart of Figure 11.10. Consequently, corrective action on the process is required by vendor management. An engineer from the vendor’s plant forms a brainstorming group comprised of workers in the Induction Hardening and Quench Department -- the department that performs case hardening on the camshafts. The brainstorming group’s aim is to determine causes for the out-of-control points in Figure 11.10. The brainstorming session’s results appear in the cause-and-effect diagram in Figure 11.11. The group determines that probable causes for out-of-control points were:

1. Point 3. Low power in the coil resulted in increased variability and less stable depth in the case hardness, as shown in cloud 1 in Figure 11.11.
2. Point 12. A temporary operator was used because the regular operator was sick, as shown in cloud 2 in Figure 11.11.
3. Point 16. The case hardness setting on the machine was incorrect, as shown in cloud 3 in Figure 11.11.
4. Point 30. Low power in the coil resulted in an out-of-control situation, as shown in cloud 1 in Figure 11.11.

Figure 11.11
Cause-and-Effect Diagram to Diagnose Reasons for Out-of-Control Points
This analysis leads vendor management to take action on the process by repairing the voltage meter on the induction hardening machine (see points 3 and 30) and training all personnel in the proper operation of the machine (see points 12 and 16). After these policies are instituted, the engineer collects 30 additional days of data and draws a new control chart, as shown in Figure 11.12 (CAMSHAFT2).

Figure 11.12
Additional Data for Camshaft Process Capability Study
We see that the vendor’s process is in statistical control, with values for $\bar{x}$ and $\bar{R}$ of 4.43 and 1.6, respectively. Recall from Chapter 8 that $\bar{R} = d_2 \sigma$, so we can calculate $\sigma$ as $\bar{R} / d_2$, where $d_2$ is found in Table B.1 in Appendix B for a subgroup of size 5. However, from the calculations in Figure 11.13, we see that $Z_{LSL} = 1.35$; that is, the process average is only 1.35 process standard deviations above the lower specification limit. Since $Z_{LSL}$ is less than 3, this indicates that the process may produce some nonconforming product. We determine the proportion of nonconforming product by examining the histogram of actual process output in Figure 11.13. As we see, one camshaft had a case hardness depth less than the lower specification limit. In other words, the process generated 0.67 percent nonconforming camshafts in the study period. It is interesting to note from the histogram that the case hardness depths are not normally distributed -- a disproportionate number of values lie near the lower specification limit. Asymmetrical situations like this could imply that there is some form of sorting prior to inspection or possibly that inspectors are accepting unsatisfactory product because they are fearful of failing to meet some production quota.

Figure 11.13
Fraction of Camshafts Out of Specification
11.3.8 The Relationships between Control Limits, Natural Limits, and Specification Limits for Variables Control Charts

*Natural Limits and Control Limits*. Natural limits are used with respect to individual observations -- and consequently on run charts. Control limits are used with respect to subgroup statistics -- and consequently on control charts. Table 11.2 shows the relationships between natural limits and control limits. We see that for $\bar{x}$ charts, if $A_2$ ($\bar{x}$ and $R$ chart) or $A_3$ ($\bar{x}$ and $s$ chart) is multiplied by the square root of the subgroup size ($\sqrt{n}$), and these new quantities ($A_2 \sqrt{n}$) and ($A_3 \sqrt{n}$) are added to and subtracted from the process average ($\bar{x}$), the control limits are transformed into natural limits. In the case of control limits for individuals charts, the control limits and the natural limits are identical because the subgroup size is one.

![Case Hardness Depth of Camshafts](image)

**Table 11.2**

**Relationship between Control Limits and Natural Limits**
for Variables Location Charts

<table>
<thead>
<tr>
<th>Chart Type</th>
<th>Control Limits</th>
<th>Natural Limits</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} ) and ( R ) chart</td>
<td>( \bar{x} \pm A_2 \bar{R} = \bar{x} \pm 3 \left( \frac{\bar{R}}{d_2} \right) )</td>
<td>( \bar{x} \pm 3(\bar{R}/d_2) )</td>
<td>If ( A_2 ) and/or ( A_3 ) are multiplied by the square root of the subgroup size ( (\sqrt{n}) ), and this new quantity is added or subtracted from ( \bar{x} )-bar-bar, the new limits are the natural limits.</td>
</tr>
<tr>
<td>( \bar{x} ) and ( s ) chart</td>
<td>( \bar{x} \pm A_3 \bar{s} = \bar{x} \pm 3 \left( \frac{\bar{s}}{c_4} \right) )</td>
<td>( \bar{x} \pm 3(\bar{s}/c_4) )</td>
<td></td>
</tr>
<tr>
<td>( \bar{x} ) with moving range chart (I-MR chart)</td>
<td>( \bar{x} \pm E_2 \bar{R} = \bar{x} \pm 3 \left( \frac{\bar{R}}{d_2} \right) )</td>
<td>( \bar{x} \pm 3(\bar{R}/d_2) )</td>
<td>Control limits and natural limits are equivalent</td>
</tr>
</tbody>
</table>

**Natural Limits and Specification Limits.** Natural limits (Voice of the Process) and specification limits (Voice of the Customer) are comparable quantities for stable processes because they are both measured with respect to the individual units of output generated by the process under study. There are four basic relationships between natural limits and specification limits for normal, stable processes. Each relationship is portrayed using a normal distribution. However, from the Empirical Rule, the assumption of normality is not necessary.

**Relationship 1.** The process's natural limits are inside the specification limits and the process is centered on nominal. This is illustrated in Figure 11.14(a).

Voice of the Customer
Nominal = 100
LSL = 60
USL = 140

Voice of the Process
Distribution = Stable
\( \mu = 100 \)
\( \sigma = 3 \)
**Figure 11.14(a)**
Relationship between Natural Limits and Specification Limits – Relationship 1

**Figure 11.14 (b)**
Relationship between Natural Limits and Specification Limits – Relationship 2

Relationship 2. The process's natural limits are inside the specification limits and the process is not centered on nominal. This is illustrated in Figure 11.14(b).

Voice of the Customer
Nominal = 100
LSL = 60
USL = 140

Voice of the Process
Distribution = Stable
$\mu = 120$
$\sigma = 3$
LNL = 111
UNL = 129
Relationship 3. The process’s natural limits are outside the specification limits and the process is centered on nominal. This is illustrated in Figure 11.14(c).

Voice of the Customer
Nominal = 100
LSL = 60
USL = 140

Voice of the Process
Distribution = Stable
μ = 100
σ = 15
LNL = 55
UNL = 145

Figure 11.14(c)
Relationship between Natural Limits and Specification Limits – Relationship 3
**Relationship 4.** The process's natural limits are outside the specification limits and the process is not centered on nominal. This is illustrated in Figure 11.14(d).

Voice of the Customer
Nominal = 100
LSL = 60
USL = 140

Voice of the Process
Distribution = Stable
μ = 120
σ = 15
LNL = 75
UNL = 165

**Figure 11.14(d)**
Relationship between Natural Limits and Specification Limits – Relationship 4
In Chapter 6 we discussed the four states of a process. [Wheeler and Chamber, 1986, pp. 12-21] Relationships 1 and 2 represent a process in its ideal state; given the same variation between relationships 1 and 2, relationship 1 is preferable. Relationships 3 and 4 represent a process in the threshold state; given the same variation between relationships 3 and 4, relationship 3 represents the more desirable situation.

Control Limits and Specification Limits. In no case should specification limits be shown on $\overline{x}$ charts. This is because control limits apply to process statistics ($\overline{x}$) and specification limits apply to individual units of process output (or some other quality characteristic). Nevertheless, specification limits are sometimes shown on control charts for Individuals. In this special case, control limits are based on subgroups of size one, and hence, on individual values.

11.3.9 Process Capability Indices for Variables Data

A common desire of many control chart users is to be able to state a process's ability to meet specifications in one summary statistic. [Kane, 1986, pp. 41-52] Such statistics are available and are called process capability indices. We use these indices to summarize internal processes as well as vendor processes.

Assumptions. All the process capability indices we will discuss here require variables data and stability of the process characteristic under study. Additionally, it is common practice to assume that this process characteristic is normally distributed.

Unfortunately, the assumption of normality is not realistic even when dealing with processes that are both stable and capable. This is because capability calculations in this situation are based on the extreme tails of the normal distribution, or the portion of the normal distribution that is beyond the specification limits. The extreme tails of the normal
distribution are mathematical quantities that rarely, if ever, characterize the real world. In the outer 5 percent of each tail, considerable discrepancies will occur between the theoretical fraction nonconforming and the actual fraction nonconforming. [Wheeler and Chambers, 1986, p. 130] Given this caveat, we may now discuss capability indices.

Indices. Four process capability indices are commonly used: \( C_p \), \( CPU \), \( CPL \), and \( C_{pk} \). They are summarized in Table 11.3.

<table>
<thead>
<tr>
<th>Index</th>
<th>Estimation Equation</th>
<th>Equation Number</th>
<th>Purpose</th>
<th>Assumptions about the Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p )</td>
<td>((USL – LSL)/\sigma)</td>
<td>11.16</td>
<td>Summarize process potential to meet acceptable tolerance s (USL – LSL)</td>
<td>Stable, Variables data, and mean = nominal</td>
</tr>
<tr>
<td>CPU</td>
<td>((USL - \bar{x})/3\sigma)</td>
<td>11.17</td>
<td>Summarize process potential to meet a one-sided LSL</td>
<td>Stable and Variables data</td>
</tr>
<tr>
<td>CPL</td>
<td>((\bar{x} - LSL)/3\sigma)</td>
<td>11.18</td>
<td>Summarize process potential to meet a one-sided USL</td>
<td>Stable and Variables data</td>
</tr>
<tr>
<td>( C_{pk} )</td>
<td>(C_p - [</td>
<td>m - \bar{x}</td>
<td>]/3\sigma)</td>
<td>11.19</td>
</tr>
</tbody>
</table>

\( C_p \). The \( C_p \) index is used to summarize a process’s ability (Voice of the Process) to meet two-sided specification limits (Voice of the Customer). In addition to the general assumptions stated above, the \( C_p \) index also assumes that the process average \( (\bar{x}) \) is centered on the nominal value, \( m \). Equation 11.16 is used to compute \( C_p \) as:

\[
C_p = \frac{(USL – LSL)}{(UNL – LNL)} = \frac{USL - LSL}{6\sigma} \quad (11.16)
\]

Recall that a process’s capability is defined to be the range in which virtually all of the output will fall; usually, this is described as plus or minus three standard deviations from the process’s mean, or within an interval of six standard deviations \( (6\sigma) \); that is, \( UNL - LNL = (\bar{x} + 3\sigma) - (\bar{x} - 3\sigma) = 6\sigma \). Consequently, if a process’s USL = UNL = \( \bar{x} + 3\sigma \) and its LSL = LNL = \( \bar{x} - 3\sigma \), the process’s capability is 1.0:

\[
C_p = \frac{(USL – LSL)}{(UNL – LNL)} = \frac{USL - LSL}{6\sigma} = \frac{(\bar{x} + 3\sigma) - (\bar{x} - 3\sigma)}{6\sigma} = \frac{6\sigma}{6\sigma} = 1.0
\]

According to the Empirical Rule, a process capability index of 1.0 indicates that a process will generate virtually all of its output within specification limits. According to the normal distribution, a process capability index of 1.0 indicates that a process will generate 99.73% of its output within specification limits. For centered processes, given the preceding
assumptions, Figure 11.15(a) shows a process with $C_p = 1.0$, indicating that the UNL = USL, and the LNL = LSL.

In the example below, $\mu = 100$, $\sigma = 10$, LNL = 70, UNL = 130, LSL = 70, USL = 130.

**FIGURE 11.15 (a)**
Process Capability Index = 1.0

<table>
<thead>
<tr>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
</tr>
<tr>
<td>0.03</td>
</tr>
<tr>
<td>0.02</td>
</tr>
<tr>
<td>0.01</td>
</tr>
<tr>
<td>0.00</td>
</tr>
</tbody>
</table>

$C_1$ histogram with normal distribution.

Figure 11.15(b) shows a process with $C_p = 2.0$, indicating that the UNL is midway between nominal and the USL, and the LNL is midway between the LSL and nominal. To state this another way, the natural limits only take up half the distance between the specification limits. According to the normal distribution, a process capability index of 2.0 indicates that a process will generate 99.9999998% of its output within specification limits.

In the example below, $\mu = 100$, $\sigma = 5$, LNL = 85, UNL = 115, LSL = 70, USL = 130.

**FIGURE 11.15 (b)**
Process Capability Index = 2.0
Figure 11.15(c) shows a process with $C_p = 0.5$, where the USL is midway between nominal and the UNL, and the LSL is midway between the LNL and nominal. To state this another way, the natural limits are twice as wide as the specification limits. According to the normal distribution, a process capability index of 0.5 indicates that a process will generate 86.64% of its output within specification limits.

In the example below, $\mu = 100$, $\sigma = 20$, LNL = 40, UNL = 160, LSL = 70, USL = 130.

**FIGURE 11.15 (c)**
Process Capability Index = 0.5
CPU. The **CPU index** is used to summarize a process's ability to meet a one-sided upper specification limit. In many situations, process owners are concerned that a process does not exceed an upper specification limit. For example, for products that can warp in only one direction, there is no LSL for warpage; the lower the warpage the better. However, there is a USL for warpage, which represents the value for warpage that will critically impair the product's ability to meet customer needs. It can also be used in situations where we want to examine one side of a two-sided specification limit.

CPU is computed using Equation 11.17 as follows:

$$\text{CPU} = (\text{USL} - \bar{x}) / (\text{UNL} - \bar{x}) = \frac{\text{USL} - \bar{x}}{3\sigma}$$

(11.17)

The CPU index measures how far the process average ($\bar{x}$) is from the upper specification limit in terms of one-sided natural tolerance limits ($3\sigma = [\text{UNL} - \bar{x}] = \{\bar{x} + 3\sigma\} - \bar{x}$).

Natural tolerances, when added and subtracted from the process mean ($\bar{x}$), yield the range in which a process is capable of operating, the process's capability, $\bar{x} \pm 3\sigma = \bar{x} \pm$ natural tolerance.

If a process's USL = UNL = $\bar{x} + 3\sigma$, the CPU is 1.0:

$$\text{CPU} = \frac{\text{USL} - \bar{x}}{3\sigma} = \frac{\text{USL} - \bar{x}}{\text{UNL} - \bar{x}} = 1.0$$

From the Empirical Rule, a CPU of 1.0 or more indicates that a process will generate virtually all of its output within the upper specification limit. From the normal distribution, a
CPU of 1.0 indicates that a process will generate 99.865% of its output within the upper specification limit.

If a process's UNL is greater than the USL, the CPU is less than 1. As \([\text{UNL} – \text{USL}]\) increases, the fraction of process output that is out of specification will increase geometrically. Conversely, if a process's UNL is less than the USL, then the CPU is greater than 1. As \(\text{USL} - \text{UNL}\) increases, the fraction of process output that is out of specification will decrease geometrically. To determine the fraction of process output that will be out of specification, we examine the histogram of process output with respect to the upper specification limit, as illustrated earlier.

CPL. The **CPL index** is used to summarize a process's ability to meet a one-sided lower specification limit. The CPL operates just like the CPU. CPL is computed using Equation 11.18 as follows:

\[
\text{CPL} = \frac{\bar{x} - \text{LSL}}{3\sigma}
\]  

(11.18)

If a process's LNL is greater than the LSL, then the CPL is less than 1. As \([\text{LNL} – \text{LSL}]\) increases, the fraction of process output that is out of specification will increase geometrically. Conversely, if a process's LNL is less than the LSL, then the CPL is greater than 1. As \(\text{LSL} - \text{LNL}\) increases, the fraction of process output that is out of specification will decrease geometrically. To determine the fraction of process output that will be out of specification, we examine the histogram of process output with respect to the lower specification limit.

\(C_{pk}\). The **\(C_{pk}\) index** is used to summarize a process's ability to meet two-sided specification limits when the process is not centered on nominal. The \(C_{pk}\) index uses the \(C_p\) index as a starting point for stating a process's capability, but it penalizes \(C_p\) if the process is not centered on nominal, \(m\). \(C_{pk}\) is computed using Equation 11.19 as follows:

\[
C_{pk} = C_p - \left\{ \frac{|m - \bar{x}|}{3\sigma} \right\}
\]  

(11.19)

The term in brackets in Equation 11.19 is always positive, and hence lowers the value of \(C_p\), which indicates that the process is less able to produce within specifications. The bracketed term is a measure of how many natural tolerance units (3\(\sigma\)) the process mean \(\bar{x}\) is from nominal \(m\). The further off-center the process, the more \(C_p\) is penalized by the bracketed factor. Hence, \(C_{pk}\) is a two-sided capability index that accounts for process centering.

A firm that exists in a **defect detection** mode will not know the process capability indices for its various processes. On the other hand, a firm operating in a **defect prevention**
mode will know the values for its various processes and will be striving for a $C_p$ or $C_{pk}$ approximately equal to or greater than 1.0. Finally, if a firm is pursuing never-ending improvement, it will be striving to move $C_p$ and $C_{pk}$ higher and higher. As $C_p$ and $C_{pk}$ become increasingly greater than 1, the specification limits from which they were computed become increasingly irrelevant.

Limitations of Capability Indices. Several potential problems exist when using the $C_p$ and $C_{pk}$ indices. First, if a process is not stable, $C_p$ and $C_{pk}$ are meaningless statistics. Second, not all processes meet the assumption of normality. Hence, the naive user of capability indices may incorrectly assess the actual fraction of process output that will be out of specification. Last, experience shows that naive users of capability indices frequently confuse $C_p$ and $C_{pk}$; they think they yield the same information about a process. Of course, this can result in a great deal of confusion.

An Example. Each process capability index discussed earlier in this chapter is calculated using the camshaft example in Figures 11.9, 11.10, and 11.12. Figure 11.10 shows the camshaft operation is out of control. After special sources of variation are removed from the process, it becomes stable, as shown in Figure 11.12. From Figure 11.12 we see that the average case hardness depth is 4.43 mm, the average range for case hardness depth is 1.60 mm, the upper specification limit is 10.5 mm, and the lower specification limit is 3.5 mm.

$$\bar{x} = 4.43 \text{ mm}$$
$$\overline{R} = 1.60 \text{ mm}; \text{ hence } \sigma = \frac{\overline{R}}{d_2} = \frac{1.60}{2.326} = 0.688 \text{ mm}$$
USL = 10.5 mm
LSL = 3.5 mm

Given these figures, the $C_p$, CPU, CPL, and $C_{pk}$ can be computed and interpreted.

$C_p$. We compute $C_p$ as

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{10.5 - 3.5}{6(0.688)} = 1.70$$

This $C_p$ indicates an extremely capable process that will almost never produce out-of-specification product. However, from Figure 11.13 we see that while virtually all the camshafts have case hardness depths within acceptable tolerances, the $C_p$ index, which assumes the process is centered, has failed to detect that the process, with an average of 4.43, is 2.57 mm off nominal ($|\bar{x} - m| = |4.43 - 7.00| = 2.57 \text{ mm}$).

CPU. We compute CPU as:

$$CPU = \frac{USL - \bar{x}}{3\sigma} = \frac{10.5 - 4.43}{3(0.688)} = 2.94$$
The CPU accurately indicates that the process is operating well within the USL of 10.5 mm.

**CPL.** We compute CPL as:

\[
CPL = \frac{\bar{x} - LSL}{3\sigma} = \frac{4.43 - 3.5}{3(0.688)} = 0.45
\]

The CPL accurately indicates that the process is not operating within the LSL of 3.5 mm.

**Cpk.** We compute Cpk as:

\[
C_{pk} = C_{pk} = C_p - \frac{|m - \bar{x}|}{3\sigma} = \frac{USL - LSL}{6\sigma} - \frac{|m - \bar{x}|}{3\sigma}
\]

\[
= \frac{10.5 - 3.5}{6(0.688)} - \frac{7.0 - 4.43}{3(0.688)} = 1.70 - 1.25 = 0.45
\]

This Cpk correctly indicates a process that will produce out-of-specification product, because, unlike Cp, it has taken into account that the process is not centered on nominal.

It should be noted that capability indices can sometimes potentially cause more problems than they can provide benefits; consequently, some practitioners recommend that they not be used.

### 11.4 Process Improvement Studies

As with process capability studies, there are two types of process improvement studies: attribute improvement studies and variables improvement studies. In the pursuit of continuous and never-ending improvement, it is natural that attribute improvement studies give way to variables improvement studies.

#### 11.4.1 Attribute Improvement Studies

Recall the data entry example discussed earlier in this chapter. The example showed that the percentage of defective data entries was stable with an average of 1.7 percent defective, and would rarely go above 4.4 percent defective. At this point, the manager decided that to improve the process further, she must study each operator individually. However, she must determine which operators to work with first. She makes a check sheet to record the number and fraction of defective lines of code by operator for December, as shown in Table 11.4.

| Table 11.4 |
| Check Sheet of Defective Lines of Code |
by Operator (12/1 - 12/31)*

<table>
<thead>
<tr>
<th>Operator</th>
<th>Frequency</th>
<th>Fraction of Defectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>2</td>
<td>.04</td>
</tr>
<tr>
<td>002</td>
<td>3</td>
<td>.06</td>
</tr>
<tr>
<td>003</td>
<td>1</td>
<td>.02</td>
</tr>
<tr>
<td>004</td>
<td>19</td>
<td>.38</td>
</tr>
<tr>
<td>005</td>
<td>0</td>
<td>.00</td>
</tr>
<tr>
<td>006</td>
<td>2</td>
<td>.04</td>
</tr>
<tr>
<td>007</td>
<td>1</td>
<td>.02</td>
</tr>
<tr>
<td>008</td>
<td>3</td>
<td>.06</td>
</tr>
<tr>
<td>009</td>
<td>17</td>
<td>.34</td>
</tr>
<tr>
<td>010</td>
<td>2</td>
<td>.04</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

* All operators produced approximately the same number of lines of code during the period under study.

Next, she constructed a c-chart for the number of defective lines of code per operator for December, shown in Figure 11.16(a). From the c-chart, she notes that operators 004 and 009 are out of control. She revises the c-chart to determine if any other operators are out of control after having removed the impact of operators 004 and 009; she finds none, as we see in Figure 11.16(b).

**Figure 11.16(a)**
Control Chart for Number of Defective Lines of Code by Operator

**Figure 11.16(b)**
Control Chart for Number of Defective Lines of Code by Operator
After Special Operators are Removed
Next, she constructs a **Pareto diagram** from Table 11.4 for the number of defects per operator. Figure 11.17 shows the Minitab Pareto diagram (the construction of a Pareto diagram using Minitab was discussed in Chapter 10, Appendix A10.1). From the Pareto diagram, she determines that 72 percent of all defective lines of code are produced by operators 004 and 009.

**Figure 11.17**
Pareto Analysis of Defective Lines of Code
The manager decides to perform separate analyses for operators 004 and 009. She begins with operator 004 by setting up a check sheet, as shown in Table 11.5, to determine the sources for operator 004’s defects.

Table 11.5
Check Sheet to Determine Defects for Operator 004 (1/1-4/30) OPERATOR4

<table>
<thead>
<tr>
<th>Cause</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transposed numbers</td>
<td>28</td>
</tr>
<tr>
<td>Wrong character</td>
<td>28</td>
</tr>
<tr>
<td>Torn document</td>
<td>4</td>
</tr>
<tr>
<td>Out of field</td>
<td>3</td>
</tr>
<tr>
<td>Data printed too lightly</td>
<td>2</td>
</tr>
<tr>
<td>Creased document</td>
<td>2</td>
</tr>
<tr>
<td>Illegible source document</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>68</strong></td>
</tr>
</tbody>
</table>

The Pareto diagram shown in Figures 11.18 indicates that 82 percent of operator 004’s defects resulted from “transposed numbers” and “wrong character.”
Subsequently, the manager forms a brainstorming group composed of three select employees to do a Cause-and-effect analysis of these two problems, as shown in Figure 11.19. The group members vote to attack both problems simultaneously. The cause-and-effect diagram leads the manager to send operator 004 to have her eyes checked. The optometrist finds that operator 004 is legally blind in her right eye. Eyeglasses correct her vision.

### Figure 11.19
Cause-and-Effect Diagrams for Operator 004

<table>
<thead>
<tr>
<th>Cause</th>
<th>Frequency</th>
<th>Percent</th>
<th>Cum %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transposed Numbers</td>
<td>28</td>
<td>41.2</td>
<td>41.2</td>
</tr>
<tr>
<td>Wrong character</td>
<td>28</td>
<td>41.2</td>
<td>82.4</td>
</tr>
<tr>
<td>Tam document</td>
<td>4</td>
<td>5.9</td>
<td>88.2</td>
</tr>
<tr>
<td>Out of field</td>
<td>3</td>
<td>4.4</td>
<td>92.6</td>
</tr>
<tr>
<td>Creased document</td>
<td>2</td>
<td>2.9</td>
<td>95.6</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>4.4</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Next, the manager collects 24 more daily samples of 200 lines of code and constructs a p chart for the fraction of defectives, as shown in Figure 11.20. From the p chart, the manager finds that operator 004’s work is now stable, has an average defective rate of 0.8 percent (8 in 1,000 lines), and rarely goes above 2.6 percent defective. The manager realizes that if she wants to improve operator 004’s performance further, she must switch from an attribute process improvement study to a variables process improvement study. Her next step is to plan her future courses of action: (1) study operator 009 and (2) review the entire department.

Figure 11.20
p Chart for Operator 004 Following Fitting with Eye Glasses
11.4.2 Variables Improvement Studies

Recall in the camshaft example that the case hardness depth was stable, with an average of 4.43 mm, and standard deviation of 0.688 mm. At this point, the engineer assigned to study the induction quench-and-harden operation decides that to improve the process further, the induction coil must be changed. The old coil is pitted and consequently emits an erratic electrical output, causing increased variability in case hardness depth between camshafts. The induction coil is changed on the evening of August 29, and then 30 more days of data are collected (August 30-October 8)

(© CAMSHAFT3 - NOTE: The row 18 data point for depth in the CAMSHAFT3 data file was changed from 6.4 to 5.9. Please change this in the data files!) and control charted, as shown in Figure 11.21.

![Xbar-R Chart of Depth](image)

The process is in statistical control, with an average case hardness depth of 5.45 mm and a standard deviation of 0.434 mm ($\bar{R} / d_2 = 1.01/2.326 = 0.434$). Note that the process has been shifted toward nominal (7.0 mm) and its unit-to-unit variation has been reduced.

Next, the process is studied to determine the number of standard deviations between the specification limits and the process average. This is done by computing $Z_{\text{LSL}}$ and $Z_{\text{USL}}$. Remember, if the process average is more than three process standard deviations from both specification limits, then according to the Empirical Rule, virtually all of the process's output will be within the specification limits.

$$Z_{\text{LSL}} = \frac{X - \text{LSL}}{\sigma} = \frac{(5.45 - 3.50)}{0.434} = 4.49$$
\[ Z_{USL} = \frac{(USL - \bar{x})}{\sigma} = \frac{(10.5 - 5.45)}{0.434} = 11.6 \]

The process is operating well within specification limits. But in the spirit of never-ending improvement (as illustrated by the Taguchi Loss Function), the engineer assigned to study the induction quench-and-harden operation should continually work to reduce unit-to-unit variation and move the process average toward nominal.

This example highlights the benefits of process improvement. The case hardness process was moved from chaos to the threshold state to the ideal state. The move from the threshold state to the ideal state resulted in:

1. Increased quality.
2. Increased productivity.
3. Lower unit cost. (It costs less to make good items than defective items because good items do not require rework.)
4. Increased price flexibility resulting from lower unit costs.
5. Increased market share resulting from increased quality and price flexibility.
6. Increased profit resulting from lower unit costs and greater market share.
7. More secure jobs for all employees.

11.5 Quality Improvement Stories

Employees trying to improve processes have found that their ideas and recommendations are more persuasive when based on data (facts), rather than opinions and guesses. The Quality Improvement (QI) story is an efficient format for employees to present process improvement studies to management. QI stories standardize quality management reports, avoid logical errors in analysis, and make reports easy for all to comprehend.

11.5.1 Relationship between QI Stories and the PDSA Cycle

A seven-step procedure is utilized to construct a QI story, following the PDSA cycle of Plan, Do, Study, and Act. The Plan phase involves three steps:

1. Select a theme for the QI story (obtain all the background information necessary to understand the selected theme, include an existing process flow chart; explain the reason for selecting the theme; and determine the organizational and departmental objective(s) that are suspected to be influenced by the theme);
2. Get a full grasp of the present situation surrounding the theme; and
3. Analyze the present situation to identify appropriate action(s) (called change concepts, or countermeasures) to the process, that is, construct a revised process flowchart that incorporates the change concepts (countermeasures).

The Do phase involves a further step: (4) testing the revised flowchart on a small scale using a planned experiment.
The Study phase involves: (5) studying, creatively thinking about, collecting, and analyzing data from the planned experiment concerning the effectiveness of the revised flowchart. Do the change concept(s) (countermeasures) reduce the difference between the Voice of the Customer and the Voice of the Process? Before and after comparisons of the experimental countermeasures' effects on the targeted department and organization objectives must be presented.

The Act phase requires two final steps: (6) determining if the revised flowchart was effective in pursuing departmental and organizational objectives. If not, we go back to the Plan stage to find other countermeasures that will be effective in pursuing departmental and organizational objectives. If the countermeasures were effective in pursuing the objectives, we either go to the Plan stage to seek the optimal settings of the countermeasure(s), or formally establish a revised best practice method and train all relevant personnel in the new best practice method. Further actions must be taken to prevent backsliding for the revised flowchart set into motion. This phase also includes: (7) identifying remaining process problems, establishing a plan for further actions, and reflecting on the positive and negative aspects of past countermeasures.

Table 11.6 relates the seven steps of the QI story to the four phases of the PDSA cycle.

<table>
<thead>
<tr>
<th>Steps of the QI Story</th>
<th>Phases of the PDSA Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Select a theme for the QI story</td>
<td>Plan Phase</td>
</tr>
<tr>
<td>2. Get a full grasp of the present situation surrounding the theme</td>
<td></td>
</tr>
<tr>
<td>3. Analyze the present situation to identify appropriate countermeasures (change concepts)</td>
<td>Do Phase</td>
</tr>
<tr>
<td>4. Set the countermeasures (change concepts) into action in a planned experiment</td>
<td>Study Phase</td>
</tr>
<tr>
<td>5. Study data concerning the effectiveness of the countermeasures (change concepts)</td>
<td></td>
</tr>
<tr>
<td>6. Establish revised standard operating procedures (best practices)</td>
<td>Act Phase</td>
</tr>
<tr>
<td>7. Establish a plan for future actions</td>
<td></td>
</tr>
</tbody>
</table>

11.5.2 Potential Difficulties

Two areas of potential difficulty when applying QI stories are qualitative (non-numerical) themes and exogenous problems. Themes that are difficult to describe with numerical values should be analyzed by focusing on the magnitude of the gap between actual performance and the desired performance. If a problem's cause (e.g., cold weather or no rain) is beyond the control of anyone in the organization, we do not conclude that it is impossible to take countermeasures to remedy the exogenous problem. Instead, we attempt to determine why there are so many occurrences of the exogenous problem in
area A versus area B, given that the areas have equal opportunities for the exogenous problem's occurrence.

11.5.3 Pursuit of Objectives

Unfortunately, QI stories will be initially selected because they are nearly complete resolutions to departmental problems and may not relate to organizational and departmental objectives. However, as employees gain experience with QI stories, they will want to select themes related to organizational and departmental objectives. A dashboard is the best vehicle for linking organizational objectives and indicators to departmental objectives and indicators. Dashboards are used to create a cascading and interlocking system of objectives and indicators, from the organizational level to the departmental level to the area level, that deploy the organization's mission statement throughout the layers of an organization. Ultimately, objectives and their indicators are linked to processes that can be improved to attain the desired levels of the indicator(s) for each objective. Dashboards are discussed in Chapter 18.

11.5.4 Quality Improvement Story Case Study

A QI story drawn from a data processing department demonstrates the role of QI stories in an organization's improvement efforts. Figure 11.22 shows the story in QI story boards 1 through 14.

![Figure 11.22](image_url)

**Quality Improvement Story**

1. Select a theme

Reduce the number of defective lines of code produced by the data entry operators

(Why do the data entry operators produce such a high percentage of defective lines of code?)
The organization’s mission mandates that every employee must base his/her decisions and actions on the following organizational objectives:

1. Pursuing continuous improvement in customer satisfaction;
2. Respecting and continuously improving all employees;
3. Establishing long-term and trusting relationships with suppliers;
4. Providing stockholders with a reasonable rate of return;
5. Being a good corporate citizen.

The data processing department’s mission mandates that every employee must base his/her decisions and actions on the following departmental objectives:

1. Recognizing that customers are both internal and external to the organization and continuously strive to improve data processing services to all customers;
2. Identifying areas in which employees require improvement and establish necessary training programs to bring about the identified improvements.

The data processing department will achieve the first departmental objective by:

1. Entering all data exactly as it appears on the source document.
2. Pursuing continuous reduction in the amount of time it takes to process a data entry job.

The manager of the data processing department realizes that the theme she selected to study is directly affected by the above objectives.
Manager realizes the importance of her intuition in pursuing her First departmental and the first and third organizational objectives.
2. Grasp of the present situation

Manager’s intuition leads her to conduct a survey to determine customer (other department’s) satisfaction with her department’s performance.

Manager constructs a list of her department’s customers:
- Administration
- Production
- Marketing

Manager constructs a questionnaire to determine customer satisfaction.

Department:___________________________________________
Supervisor:_____________________________________________

(1) Do you feel that the error rate that data entry provides your department is:
- unsatisfactory [ ]
- satisfactory [ ]
- excellent [ ]

(2) Approximately what percent of the data entry error your department receives from our department contain errors attributable to our department?___________%
Questionnaires were sent to all of the departments and all of the departments responded. Analysis of the questionnaires yielded the following results:

Findings:
- Do you feel that the error rate that data entry provides your department is:
  - unsatisfactory? (72%)
  - satisfactory? (20%)
  - excellent? (8%)
- Approximately 2% of the data entry errors received by the various departments contain errors attributable to the data processing department.
Due to customer dissatisfaction, the manager decided to collect data concerning the daily proportion of defective entry errors.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of Lines inspected</th>
<th>Number of defective lines</th>
<th>Fraction of defective lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>5</td>
<td>.025</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>6</td>
<td>.030</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>14</td>
<td>.070</td>
</tr>
<tr>
<td>9</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>11</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
<td>8</td>
<td>.040</td>
</tr>
<tr>
<td>13</td>
<td>200</td>
<td>2</td>
<td>.010</td>
</tr>
<tr>
<td>14</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>15</td>
<td>200</td>
<td>7</td>
<td>.035</td>
</tr>
<tr>
<td>16</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>17</td>
<td>200</td>
<td>3</td>
<td>.015</td>
</tr>
<tr>
<td>18</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td>19</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
<td>0</td>
<td>.000</td>
</tr>
<tr>
<td>21</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>22</td>
<td>200</td>
<td>15</td>
<td>.075</td>
</tr>
<tr>
<td>23</td>
<td>200</td>
<td>4</td>
<td>.020</td>
</tr>
<tr>
<td>24</td>
<td>200</td>
<td>1</td>
<td>.005</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>4,800</strong></td>
<td><strong>102</strong></td>
<td></td>
</tr>
</tbody>
</table>

Finding:
The data entry operation is in a state of chaos; it produces an unknown proportion of defectives per day.
3. Analysis of the present situation

Process must be stabilized. Hence causes for days 8 and 22 must be found and policy must be set to prevent them from reoccurring.

The manager reviewed her daily comments concerning any unusual events that occurred on days 8 and 22.

Log sheet for data entry operation

<table>
<thead>
<tr>
<th>Day</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>untrained operator used for a rush job.</td>
</tr>
<tr>
<td>22</td>
<td>ran out of toner From usual vendor</td>
</tr>
</tbody>
</table>

Findings:
- Policy needed For training new operators used for rush jobs
- Inventory policy needed to set safety stock level

PLAN ESTABLISHED
Manager sets the policies into motion on a trial basis. She collects more data and checks to see if the process is stable and improved.

The countermeasures set into motion on a trial basis were effective in pursuit of the first data processing departmental objective; the data entry process is stable and generates 1.7% defective entries on average per day (down from an unstable 2.1% per day) Rarely will the daily average entries rise above 4.4% (down from 5.2%).
The manager decides that a random sample of 200 lines per month will be drawn from every operator's output. These samples will be analyzed so that appropriate actions can be taken to prevent any backsliding in areas that have been improved.

The department manager will continue to study the process to seek ways to lower the defect rate.

Manager realizes that to improve the data entry process she must conduct a separate study for each operator.
### 2. Grasp the present situation

Check sheet of defective entries by operator (All operators produced approximately the same number of lines of code during the period under study.[10/1-10/31]

<table>
<thead>
<tr>
<th>Operator</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>004</td>
<td>++++</td>
<td>19</td>
</tr>
<tr>
<td>005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>009</td>
<td>++++</td>
<td>17</td>
</tr>
<tr>
<td>010</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td><strong>50</strong></td>
</tr>
</tbody>
</table>
What percentage of the departmental errors are caused by operators 004 and 009?

Finding: 72% of all defective lines are produced by operators 004 and 009!!!

<table>
<thead>
<tr>
<th>Operator</th>
<th>Freq.</th>
<th>%</th>
<th>Cum. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>19</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>34</td>
<td>72</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>78</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>4</td>
<td>92</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>4</td>
<td>96</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>98</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Pareto Analysis of defective lines by operator (10/1-10/31)

Operators are stable without operators 004 and 009.

Operators 004 and 009 are out of the lines system.

Finding: 72% of all defective lines are produced by operators 004 and 009!!!
Manager decides to study operators 004 and 009; she begins with operator 004.

### Checklist to determine the sources of operator 004’s defective lines (Jan. – Apr.)

<table>
<thead>
<tr>
<th>Major causes of Defective lines</th>
<th>Month</th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transposed numbers</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>Out of field</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Wrong character</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>Data printed too lightly</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Torn document</td>
<td>1</td>
<td>1</td>
<td></td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Creased document</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Illegible source document</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>15</td>
<td>20</td>
<td>16</td>
<td>17</td>
<td>68</td>
</tr>
</tbody>
</table>
Pareto Analysis to determine major causes of defective lines of code
For operator 004 (Jan-Apr)

<table>
<thead>
<tr>
<th>Major causes of defective lines</th>
<th>Freq.</th>
<th>%</th>
<th>Cum. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transposed numbers</td>
<td>28</td>
<td>41.2</td>
<td>41.2</td>
</tr>
<tr>
<td>Wrong character</td>
<td>28</td>
<td>41.2</td>
<td>82.4</td>
</tr>
<tr>
<td>Torn document</td>
<td>4</td>
<td>5.9</td>
<td>88.3</td>
</tr>
<tr>
<td>Out of field</td>
<td>3</td>
<td>4.4</td>
<td>92.7</td>
</tr>
<tr>
<td>Data printed too lightly</td>
<td>2</td>
<td>2.9</td>
<td>95.6</td>
</tr>
<tr>
<td>Creased document</td>
<td>2</td>
<td>2.9</td>
<td>98.5</td>
</tr>
<tr>
<td>Illegible source document</td>
<td>1</td>
<td>1.5</td>
<td>100.0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>68</strong></td>
<td><strong>100.0</strong></td>
<td></td>
</tr>
</tbody>
</table>

Findings:
Transposed numbers and wrong character account for 82.4% of operator 004’s defective lines of code.

Pareto Diagram to determine causes of defective lines for operator 004
Manager forms a group for a brainstorming session concerning how to resolve operator 004’s problems with transposed numbers and wrong characters.

Brainstorming group votes to work on both problems simultaneously by a unanimous vote of the group.

- Materials
- Machines
- Environment
- Poor eyesight
- Methods
- Personnel
- Transposed numbers and wrong characters

PLAN ESTABLISHED

QI STORY BOARD 11

4. Set the countermeasures in motion

Operator 004 is sent to have her eyes examined by an optometrist. She needs and receives eyeglasses.
Manager collects 25 additional daily samples of 200 lines of code each to determine the effect of operator 004’s eyeglasses on her defective rate.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number Defective</th>
<th>Total Lines of Code</th>
<th>Proportion Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>200</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>200</td>
<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>200</td>
<td>0.010</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>200</td>
<td>0.010</td>
</tr>
<tr>
<td>40</td>
<td>5000</td>
<td></td>
<td>0.008</td>
</tr>
</tbody>
</table>

p charts comparing the proportion of defective lines produced by the "average" operator before improvement efforts with the proportion of defective lines produced by operator 004 after improvement efforts.

Finding: Operator 004 is stable and producing 8 defective lines per 1,000. Rarely will her defect rate fo above 2.6 per 1,000. The countermeasure taken with operator 004 is effective in the pursuit of the first data processing department objective.
6. Standard operating procedure

The manager establishes a formal procedure by sending operator 004 for glasses.

The manager formally establishes a policy stating that all operators must have their eyes examined yearly and provide evidence of said examination. If any operator needs glasses, she/he will receive them. This policy should prevent backsliding in improvement efforts due to poor eyesight.

7. Plan for future action

<table>
<thead>
<tr>
<th>Phase 2</th>
<th>Check Progress of Entire Department</th>
<th>When will future plan Be carried out?</th>
<th>Who will Carry out Plan?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>May</td>
<td>Jun</td>
</tr>
</tbody>
</table>
The QI story goes through two iterations of the PDSA cycle; nevertheless, a never-ending set of PDSA iterations will follow as the data processing department pursues continuous improvement in its daily work. The first iteration of the PDSA cycle focuses on all data entry operators in the data processing department. In this iteration of the PDSA cycle, Select a Theme is presented in QI story board 1; this includes showing the background of theme selection and the reason for selecting the theme in relation to the organization's and department's objectives. A Grasp of the Present Situation is presented in QI story board 2. An Analysis of the Present Situation, in QI story board 3, is performed to determine appropriate countermeasures that pursue the theme and the organization and department objectives. Set the Countermeasures into Motion on a trial basis is presented in QI story board 4. The Effectiveness of the Countermeasures on the theme and the organization and department objectives is measured as shown in QI story board 5. Standard Operating Procedure is set that formalizes the countermeasures and prevents backsliding in QI story board 6. A Plan for Future Actions is presented in QI story board 7.

The second iteration of the PDSA cycle focuses attention on an individual data entry operator. In this iteration, Select a Theme is accomplished when the data processing manager realizes that future process improvements will require her to identify and train operators whose performance is out of control on the high side, shown in QI story board 8. In this iteration of the PDSA cycle, a Grasp of the Present Situation determines that data entry operators 004 and 009 are out of control on the high side and why operator 004 is out of control on the high side; this is presented in QI story board 9. An Analysis of the Present Situation, in QI story board 10, determines the countermeasures necessary to improve operator 004's work. The manager Sets the Countermeasures into Motion, shown in QI story board 11. The positive Effectiveness of the Countermeasures on operator 004 and on the organization and department objectives is confirmed, as shown in QI story board 12. Standard Operating Procedure is set, which formalizes the countermeasure to all operators and prevents backsliding, in QI story board 13. Finally, a Plan for Future Action is specified in QI story board 14.

11.6 Summary

An organization’s quality consciousness can be better understood by examining the types of specifications it uses in production and service. If a firm uses individual unit and/or AQL specifications as guidelines to separate good product/service from bad, the firm is operating in a defect detection mode. If a firm uses individual unit specification as guidelines to determine the percentage of its output that is out-of-specification so that the difference between customer needs and process performance can be decreased, the firm has advanced to a defect prevention mode. Finally, if a firm uses distribution or performance specifications in its never-ending pursuit of total process improvement, it is operating in a never-ending improvement mode. The goals of never-ending improvement are the constant reduction of unit-to-unit variation and the movement of the process average toward nominal. Conformance to technical specifications (Zero Defects) is not an acceptable form of quality consciousness. Ultimately, it will result in entropy and the deterioration of the process.
Process capability studies determine if a process is unstable, investigate any sources of instability and determine their causes, and take action to eliminate these sources of instability. After all sources of instability have been eliminated, the natural behavior of the process is called its process capability. We use types of process capability studies: attribute studies and variables studies. For each, we consider data requirements and possible actions that can be taken on the process as a result of the process capability study. For variables process capability studies, we consider the relationship between control limits, natural limits, and specification limits, and process capability indices.

Quality improvement stories provide an efficient format for employees to present process improvement studies to management; they standardize quality management reports, avoid logical errors in analysis, and make reports easy for all to comprehend.

**EXERCISES**

11.1 What is the basic function of a performance specification?

11.2 Explain the purpose and describe the construction of the three types of technical specifications: individual unit specifications, acceptable quality level specifications, and distribution specifications.

11.3 In an assembly operation, steel sheet A is glued onto steel sheet B to create a double-sheet thickness. Assume that the glue has no discernible thickness and that the unit-to-unit variation in thickness for both types of steel sheets is stable over time. The resulting thickness of the combined steel sheets is the quality characteristic of interest. The following process statistics have been collected concerning both types of steel sheets:

<table>
<thead>
<tr>
<th></th>
<th>Steel Sheet A</th>
<th>Steel Sheet B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.50 inches</td>
<td>4.75 inches</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.25 inches</td>
<td>0.50 inches</td>
</tr>
</tbody>
</table>

a. Compute the mean of the double-sheet thickness.
b. Compute the standard deviation of the double-sheet thickness.

11.4 Rectangular sheets of material are produced in an assembly operation. Their dimensions are 9.0 inches in width by 14.0 inches in length. Assume that unit-to-unit variations among the widths and lengths of the rectangular sheets are stable over time. The area of the sheets is the quality characteristic of interest. The following process statistics have been collected for the widths and lengths of the rectangular sheets:

<table>
<thead>
<tr>
<th></th>
<th>Width</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.0 inches</td>
<td>14.0 inches</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.10 inches</td>
<td>0.40 inches</td>
</tr>
</tbody>
</table>
a. Compute the mean area of the rectangular sheets.
b. Compute the standard deviation of the area of the rectangular sheets.

11.5 The ABC Company produces steel tubes. The steel tube process is a stable cut-to-length operation that generates tubes that have a mean of 12.00 inches and standard deviation of 0.10 inches. The XYZ Company wishes to buy tubes from the ABC Company. The XYZ Company requires steel tubes between 11.77 inches and 12.23 inches in length.

b. Compute $CPU$.
c. Compute $CPL$.
d. Compute $C_{pk}$.
e. Compute $Z_{LSL}$.
f. Compute $Z_{USL}$.
g. Compare and contrast the preceding capability indices with respect to their ability to explain the capability of the ABC Company's steel tube process.
h. Discuss the managerial implications of the capability indices you computed in parts (a) through (f).

11.6 The LMN Company wishes to buy tubes from the ABC Company. The LMN Company requires steel tubes 11.95 inches long with a tolerance of 0.30 inches.

b. Compute $CPU$.
c. Compute $CPL$.
d. Compute $C_{pk}$.
e. Compute $Z_{LSL}$.
f. Compute $Z_{USL}$.
g. Compare and contrast the preceding capability indices with respect to their ability to explain the capability of the ABC Company's steel tube process.
h. Discuss the managerial implications of the capability indices you computed in parts (a) through (f).

11.7 The Arco Company produces plastic containers. The plastic container process is a stable operation that generates containers with a mean volume of 12,500.00 cubic inches and standard deviation of 10.00 cubic inches. The Beta Company wishes to buy plastic containers from the Arco Company. The Beta Company requires plastic containers with a volume between 12,495.00 cubic inches and 12,545.00 cubic inches.

b. Compute $CPU$.
c. Compute $CPL$.
d. Compute $C_{pk}$.
e. Compute $Z_{LSL}$.
f. Compute $Z_{USL}$.
g. Compare and contrast the preceding capability indices with respect to their ability to explain the capability of the Beta Company's plastic container process.
h. Discuss the managerial implications of the capability indices you computed in parts (a) through (f).

11.8 The Largo Corporation wishes to buy plastic containers from the Arco Company. The Largo Corporation requires plastic containers with a volume of 12,495.00 cubic inches and a tolerance of 20.00 cubic inches.

b. Compute $CPU$.
c. Compute $CPL$.
d. Compute $C_{pk}$.
e. Compute $Z_{LSL}$.
f. Compute $Z_{USL}$.
g. Compare and contrast the preceding capability indices with respect to their ability to explain the capability of the Beta Company's plastic container process.
h. Discuss the managerial implications of the capability indices you computed in parts (a) through (f).

11.9 How do you determine a process's capability, given that the only information available comes from an attribute process capability study?

11.10 Answer the following questions concerning QI stories:
a. Discuss the purpose of a QI story.
b. List the seven steps in a QI story.
c. Explain the relationship between the seven steps in a QI story and the four stages of the PDSA cycle.

11.11 Define capability of a process in statistical terms. Consider the Empirical Rule in your definition.

11.12 Answer the following questions concerning the data requirements for process capability studies:
a. Discuss the data requirements to conduct an attribute process capability study. Consider the number of time periods and the number of subgroups per time period.
b. Discuss the data requirements to conduct a variables process capability study. Consider the number of time periods and the number of subgroups per time period.
REFERENCES


